Weakly-nonlinear Prandtl model for simple slope flows

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Kranjska Gora, ICAM2013, June
After E. Ekhart 1948

After C.D. Whiteman 2000

Weakly-nonlinear Prandtl model
INTRO

Motivation & Overview

PRANDTL MODEL

Classic, Now weakly-nonlinear, $K(z)_{WKB}$ approximation

RESULTS & DISCUSSION

Comparison with Pasterze glacier wind data, recommendations to climate modelers to parameterize ...
Typical reg. clim. simulation, RegCM, 1989-1998, $dx \approx 50$ km

Winter & summer $T_2m$ "errors" 1st order turb. scheme

Same but with H.O.C. turbulence scheme
Clim. simulations & obs – cont’d - “errors” for wind simulations

Winter & summer wind magn. errors, 1st order turb. scheme

Same but with H.O.C. turbulence scheme
Many avenues to cross…

- Better resolution, numerics, SEB, microphysics, clouds, precipitation, ABL…
- Improving ABL schemes over complex terrain, especially those for the SABL
- …Thermally driven flows should be better treated in climate models…start with Prandtl “kata-ana” model…
CLASSIC PRANDTL MODEL

- Stationarity for \((\theta, U)\) after \(t > 1-1.5T, T = \frac{2\pi}{N\sin(\alpha)}\)
- PDE → ODE
- \((\alpha, \Gamma, C) \Rightarrow \theta(z), U(z) = \text{exponential-complex decay}\)

\[U:\]
\[0 = \frac{g}{\theta_0} \sin(\alpha)\theta + \frac{d}{dz} \left( \Pr K \frac{dU}{dz} \right)\]

\[\theta:\]
\[0 = -\Gamma \sin(\alpha)U + \frac{d}{dz} \left( K \frac{d\theta}{dz} \right)\]

Weakly-nonlinear Prandtl model
Modified: weakly-nonlinear Prandtl model

\[ u: \quad 0 = g \frac{\theta}{\theta_0} \sin(\alpha) + KPr \frac{\partial^2 u}{\partial z^2} \]

\[ \theta: \quad 0 = -\left( \Gamma + \varepsilon \frac{\partial \theta}{\partial z} \right) u \sin(\alpha) + K \frac{\partial^2 \theta}{\partial z^2}, \quad 0 \leq \varepsilon \ll 1 \]

\[ u_{tot} = u_0 + \varepsilon u_1 + \varepsilon^2 u_2 + \ldots \quad \theta_{tot} = \theta_0 + \varepsilon \theta_1 + \varepsilon^2 \theta_2 + \ldots \]
\[ \varepsilon_1^1: \]

\[ \frac{\partial^4 u_1}{\partial z^4} + u_1 \frac{g \Gamma \sin^2(\alpha)}{\Theta_0 Pr K^2} = -\frac{g \sin^2(\alpha)}{\Theta_0 Pr K^2} u_0 \frac{\partial \theta_0}{\partial z} \]

\[ \frac{\partial^4 \theta_1}{\partial z^4} + \theta_1 \frac{g \Gamma \sin^2(\alpha)}{\Theta_0 Pr K^2} = \frac{\sin(\alpha)}{K} \frac{\partial^2}{\partial z^2} \left( u_0 \frac{\partial \theta_0}{\partial z} \right) \]

- This is again damped oscillator but now forced by the \( \varepsilon^0 \) state.

- Whole solution = \( 0^{TH} + 1^{ST} \) order \( \Leftrightarrow \) exp-decay with various coeff’s … each \((u_1, \theta_1)\) has 5 terms… (see Toni’s diploma work)
Estimating the small parameter \( \varepsilon \), … expecting \( \varepsilon \leq O(0.1) \)

\[
\max(\varepsilon) \leq \frac{\Gamma}{h_p \exp(-\pi)} \]

Thermodyn. Eqn \( \rightarrow \) Numerical experiments \( \rightarrow \)

Here for Pasterze data: \( \varepsilon = 0.005 \)
WKB approximation - gradual $K(z)$

Solutions keep their original structure as from the const. coeff.

ODE solutions but now with local values (integrated from below if previously multiplied by $z$) $\Leftrightarrow$ 0$^{TH}$ order WKB

Chosen $K(z) = K_0 \frac{z}{h} \exp\{-\frac{z^2}{2h^2}\}$, or similar, $h > LLJ$

details in JAS’01, QJRMS’01, ACP’10, etc.
PASTEX’94 data (8 level 13 m tower & balloon) vs. Prandtl model

Weakly-nonlinear Prandtl model
Conclusions

- **Climate models poorly treat diurnal mnt. cycle, coastal & mnt. flows, related precip…**

- **Slope flows parameterization should be included in climate models since the models’ poor Δz, Δx**

- **Modified Prandtl model allows for stronger & sharper inv. during weaker winds, large near-surface Ri_{grad} # – missing in most of NWP & climate models**

- **Chances for diploma works, PhD ↔ parameterization ↔ postdocs, senior projects – climate modeling…**
Spare slides

• Eqns!
\[
\frac{4}{h_p^4} = \frac{g \Gamma \sin^2(\alpha)}{\Theta_0 \Pr K^2}
\]

\[
h_p = \frac{\sqrt{2}}{\sigma}, \quad \sigma = \sqrt{\frac{N \sin(\alpha)}{\Theta_0 \Gamma \Pr^{1/2}}}, \quad \mu = \sqrt{\frac{g}{\Theta_0 \Gamma \Pr}}, \quad N^2 = \frac{\Gamma g}{\Theta_0}
\]

\[
\theta_{1,\text{hom}} \sim \exp(\lambda \xi) \quad \xi = \frac{2z}{h_p}
\]

\[
\theta_{1,\text{hom}} = G_1 \exp\left(-\frac{\xi}{2}\right)\sin\left(\frac{\xi}{2}\right) + G_2 \exp\left(-\frac{\xi}{2}\right)\cos\left(\frac{\xi}{2}\right)
\]

\[
\theta_{1,\text{part}} = A_1 \exp(-\xi)\cos(\xi) + B_1 \exp(-\xi)\sin(\xi) + C_1 \exp(-\xi)
\]

\[
\theta_{\text{tot}}(z) = \theta_0(z) + \varepsilon \theta_1(z) \bigg|_{z \to 0} = C
\]

\[
u_{\text{tot}}(z) = u_0(z) + \varepsilon u_1(z) \bigg|_{z \to 0} = 0
\]

\[
\theta_1(z = 0) = u_1(z = 0) = 0
\]

\[
\theta_1(z \to \infty) = u_1(z \to \infty) = 0
\]

Weakly-nonlinear Prandtl model
Final sol. for $\varepsilon^1$ part (1st order), while the whole solution = 0th + 1st order:

$$\theta_1(z) = \theta_A \exp\left(-\frac{z}{h_p}\right)\left[-\frac{1}{15}\sin\left(\frac{z}{h_p}\right) - \frac{1}{6}\cos\left(\frac{z}{h_p}\right)\right]$$

$$+ \theta_A \exp\left(-\frac{2z}{h_p}\right)\left[\frac{1}{15}\sin\left(\frac{2z}{h_p}\right) + \frac{1}{15}\cos\left(\frac{2z}{h_p}\right) + \frac{1}{10}\right]$$

$$u_1(z) = u_A \exp\left(-\frac{z}{h_p}\right)\left[-\frac{1}{3}\sin\left(\frac{z}{h_p}\right) + \frac{2}{15}\cos\left(\frac{z}{h_p}\right)\right]$$

$$+ u_A \exp\left(-\frac{2z}{h_p}\right)\left[-\frac{1}{30}\cos\left(\frac{2z}{h_p}\right) + \frac{1}{30}\sin\left(\frac{2z}{h_p}\right) - \frac{1}{10}\right]$$

$\theta_A = \frac{h_p C^2 \mu \sin(\alpha)}{K}$, $u_A = \frac{C^2 \mu}{h_p \Gamma}$

Weakly-nonlinear Prandtl model