Weakly-nonlinear Prandtl model for simple slope flows

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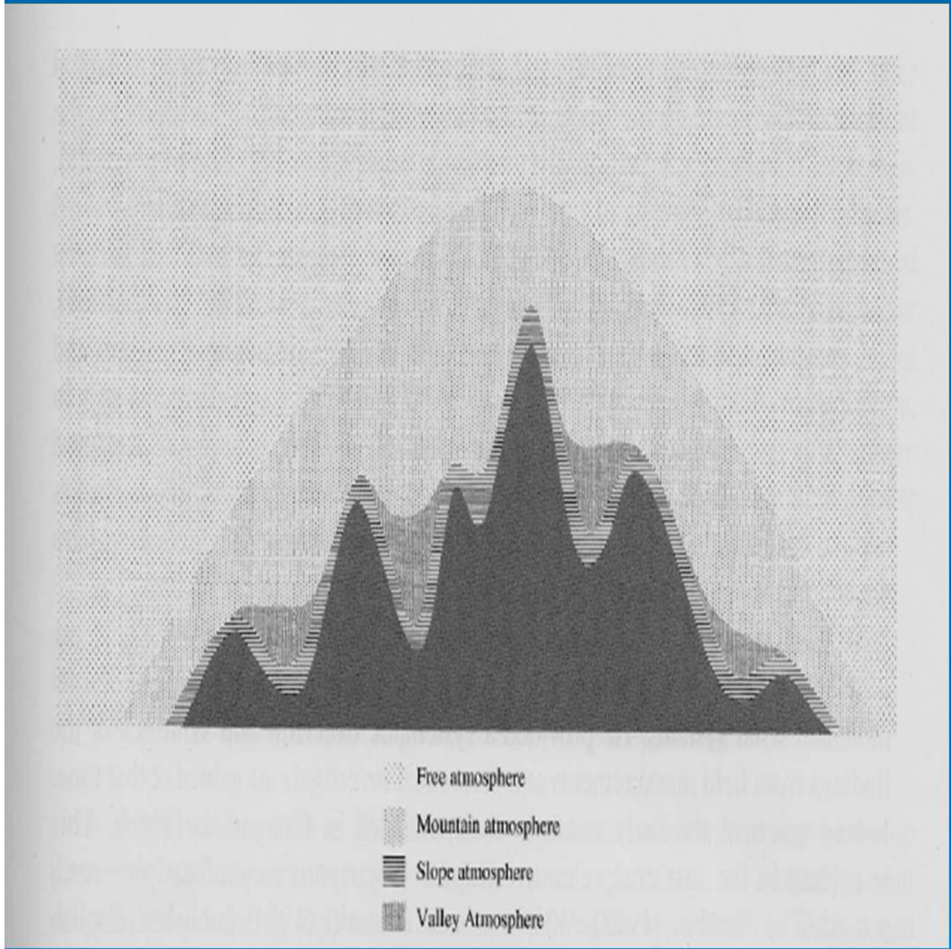
…perspective…



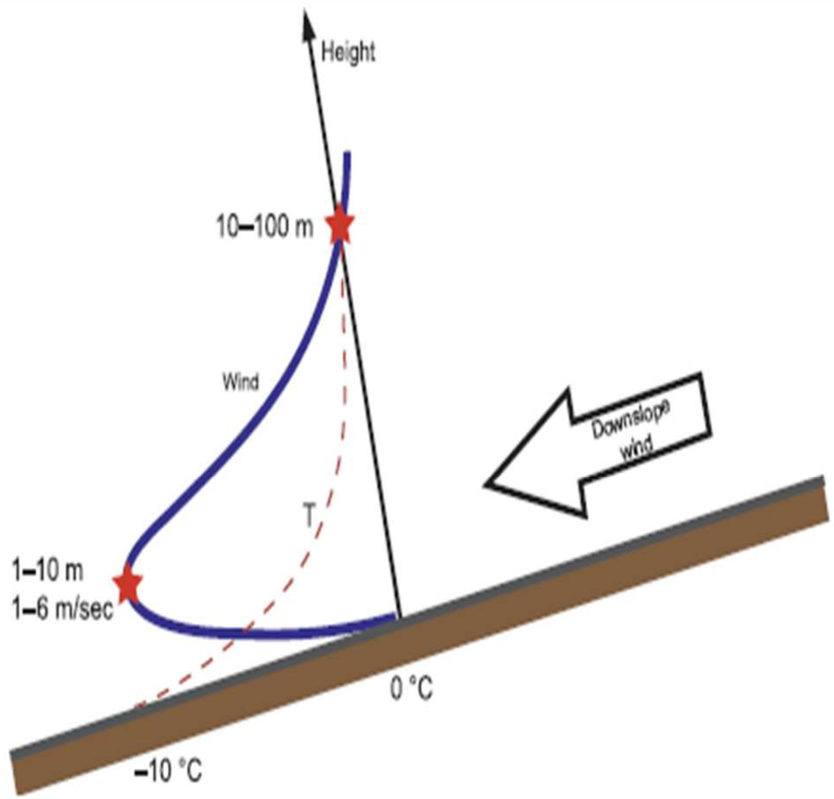


Fig. 2. Illustration of typical wind and temperature profiles in katabatic wind layer (adapted from Whiteman 2000).

After E. Ekhart 1948

After C.D. Whiteman 2000

CONTENT

- INTRO

  Motivation & Overview

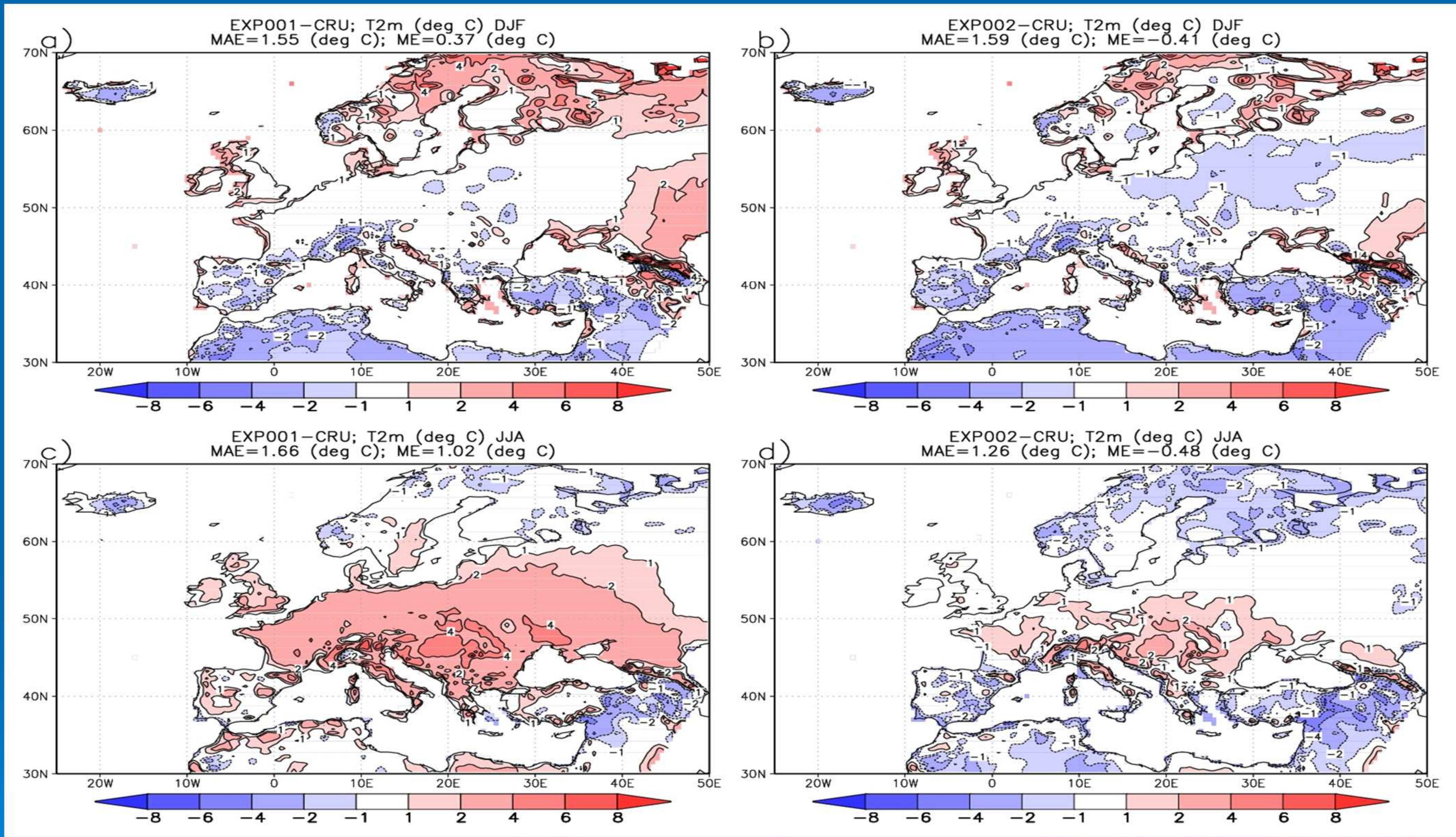
- PRANDTL MODEL

  Classic, Now weakly-nonlinear, $K(z)_{WKB}$ approximation

- RESULTS & DISCUSSION

  Comparison with Pasterze glacier wind data, recommendations to climate modelers to parameterize …

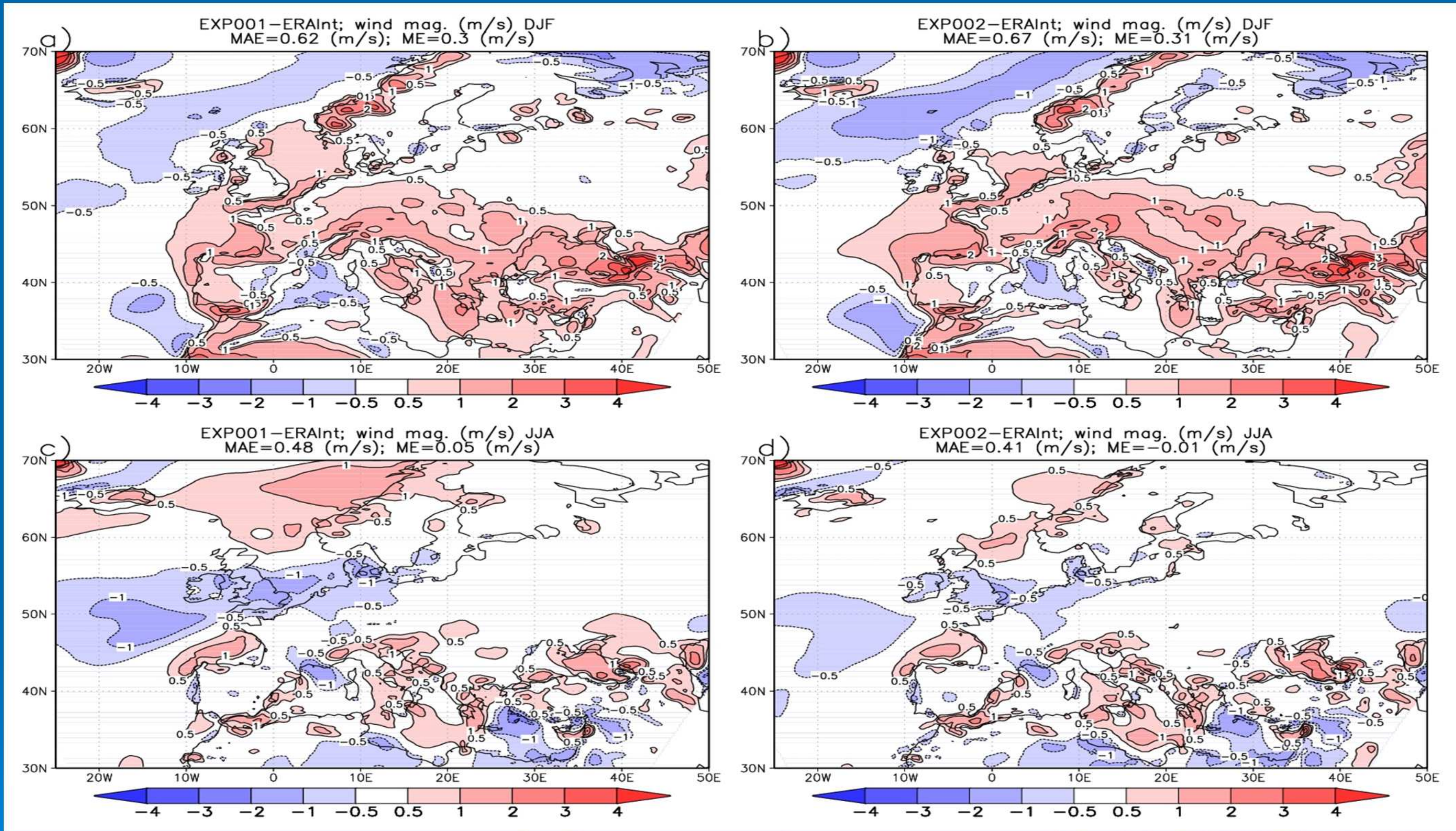
Typical reg. clim. simulation, RegCM, 1989-1998, dx ≈ 50 km



Winter & summer **T2m <u>“errors”</u>** st order turb. scheme

Same but with H.O.C. turbulence scheme

Clim. ***simulations*** *&* ***obs*** *– cont'd - "errors" for wind simulations*



Winter & summer **wind magn.** errors, st order turb. scheme

Same but with H.O.C. turbulence scheme

Many avenues to cross…

- *Better resolution, numerics, SEB, micro-physics, clouds, precipitation, ABL…*
- *Improving ABL schemes over complex terrain, especially those for the SABL*
- *…Thermally driven flows should be better treated in climate models…start with Prandtl “kata-ana” model…*

CLASSIC PRANDTL MODEL

- *Stationarity for (θ , **U**) after t >* ***1-1.5T, T = 2π/(Nsin(α))***
- *PDE $\longrightarrow$ ODE*
- ***(α, Γ, C) => θ(z), U(z)*** *= exponential-complex decay*

U:

$$0 = \frac{g}{\theta_0}\sin(\alpha)\theta + \frac{d}{dz}\left(\Pr K \frac{dU}{dz}\right)$$

θ:

$$0 = -\Gamma \sin(\alpha)U + \frac{d}{dz}\left(K \frac{d\theta}{dz}\right)$$

Modified: weakly-nonlinear Prandtl model

u:

$$0 = g\frac{\theta}{\Theta_0} sin(\alpha) + KPr\frac{\partial^2 u}{\partial z^2}$$

θ:

$$0 = -\left(\Gamma + \boldsymbol{\varepsilon}\frac{\boldsymbol{\partial\theta}}{\boldsymbol{\partial z}}\right) u\, sin(\alpha) + K\frac{\partial^2 \theta}{\partial z^2}, \qquad 0 \le \varepsilon \ll 1$$

- $u_{tot} = u_0 + \varepsilon u_1 + \varepsilon^2 u_2 + ..$
- $\theta_{tot} = \theta_0 + \varepsilon\theta_1 + \varepsilon^2\theta_2 + ..$

ε^1:

- $$\frac{\partial^4 u_1}{\partial z^4} + u_1 \frac{g\Gamma sin^2(\alpha)}{\Theta_0 Pr K^2} = -\frac{g sin^2(\alpha)}{\Theta_0 Pr K^2} u_0 \frac{\partial \theta_0}{\partial z}$$

- $$\frac{\partial^4 \theta_1}{\partial z^4} + \theta_1 \frac{g\Gamma sin^2(\alpha)}{\Theta_0 Pr K^2} = \frac{sin(\alpha)}{K} \frac{\partial^2}{\partial z^2}\left(u_0 \frac{\partial \theta_0}{\partial z}\right)$$

-This is again damped oscillator but now forced by the ε^0 state.

-Whole solution = 0^{TH} + 1^{ST} order ⇔ exp-decay with various coeff's … each (u_1, θ_1) has 5 terms…(see Toni's diploma work)

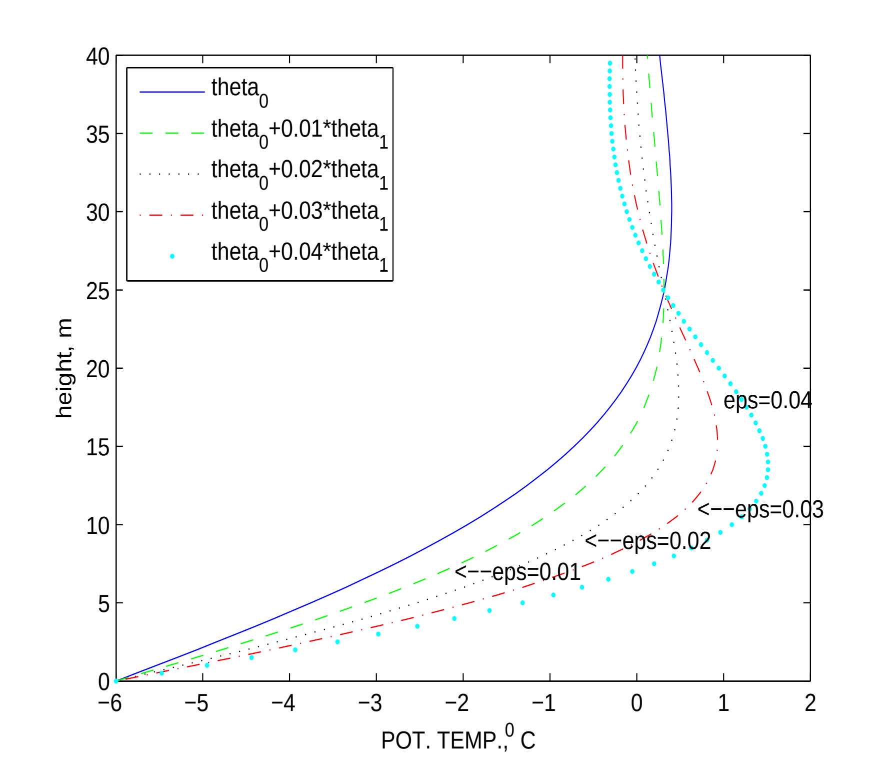
- *Estimating the small parameter ε, … expecting ε ≤ O(0.1)*

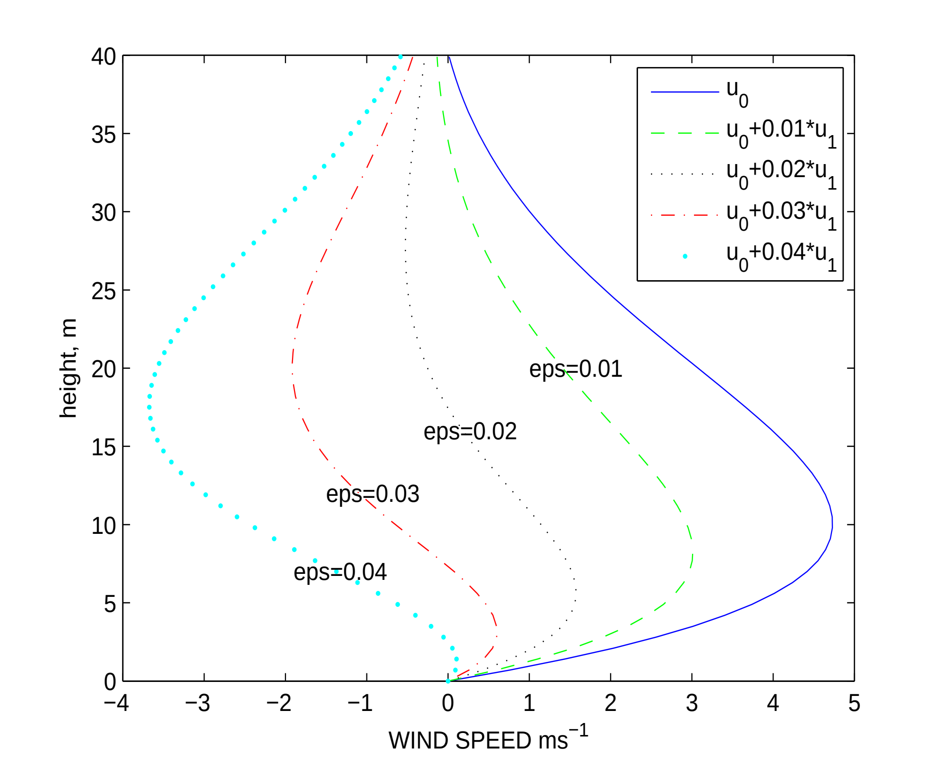
Thermodyn. Eqn →

$$\max(\varepsilon) \le \frac{\Gamma}{\left|\frac{C}{h_p}\exp(-\pi)\right|}$$

… Numerical experiments →

$$\max(\varepsilon) \le \frac{\frac{\Gamma}{15}}{\left|\frac{C}{h_p}\exp(-\pi)\right|}$$





here for Pasterze data: ε = 0.005

Weakly-nonlinear Prandtl model

WKB approximation - gradual K(z)

Solutions keep their original structure as from the const. coeff.

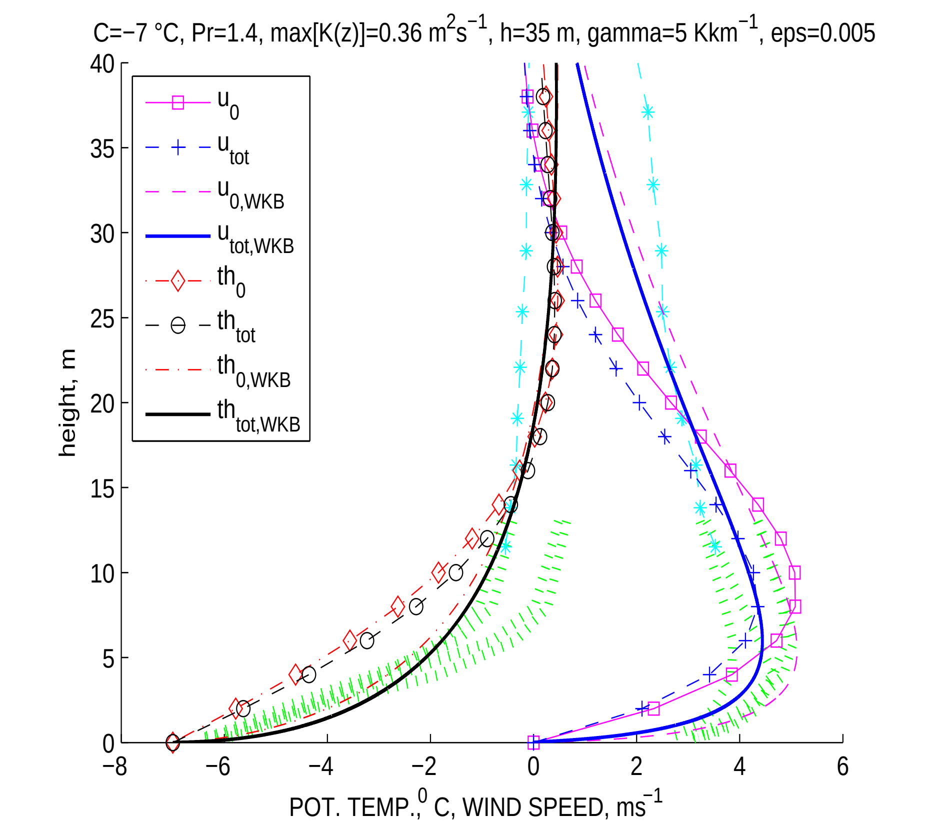
ODE solutions but now with local values (integrated from

below if previously multiplied by z) ⇔ 0^{TH} order WKB

Chosen $K(z) = K_0\, z/h\, \exp\{-z^2/(2h^2)\}$, or similar, $h >$ LLJ

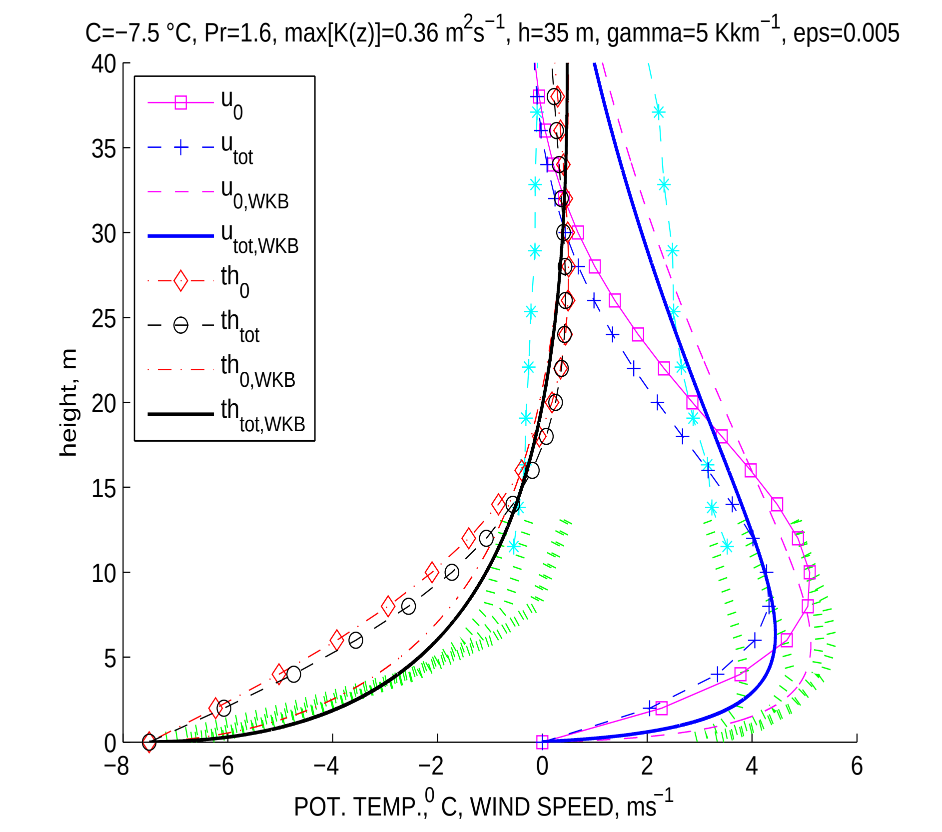
details in JAS'01, QJRMS'01, ACP'10, etc.

PASTEX'94 data (8 level 13 m tower & balloon) vs. Prandtl model



tower = green - hatched

balloon = turquoise - star-dashed



Morning ↑ *Afternoon →*

Conclusions

- *Climate models poorly treat diurnal mnt. cycle, coastal & mnt. flows, related precip….*

- *Slope flows parameterization should be included in climate models since the models' poor Δz, Δx*

-Modified Prandtl model allows for stronger & sharper inv. during weaker winds, large near-surface Ri_{grad} # – missing in most of NWP & climate models

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-Chances for diploma works, PhD ↔ parameterization ↔ postdocs, senior projects – climate modeling…

Spare slides

- Eqns!

$$\frac{4}{h_p^4} = \frac{g\Gamma \sin^2(\alpha)}{\Theta_0 \Pr K^2}$$

$$h_p = \frac{\sqrt{2}}{\sigma},\ \sigma = \sqrt{\frac{N \sin(\alpha)}{K \Pr^{1/2}}},\ \mu = \sqrt{\frac{g}{\Theta_0 \Gamma \Pr}},\ N^2 = \frac{\Gamma g}{\Theta_0}$$

$$\theta_{1,\mathrm{hom}} \sim \exp(\lambda \xi)$$

$$\xi = 2z / h_p$$

$$\theta_{1,\mathrm{hom}} = \mathrm{G}_1 \exp(-\frac{\xi}{2}) \sin(\frac{\xi}{2}) + \mathrm{G}_2 \exp(-\frac{\xi}{2}) \cos(\frac{\xi}{2})$$

$$\theta_{1,part} = A_1 \exp(-\xi) \cos \xi + B_1 \exp(-\xi) \sin \xi + C_1 \exp(-\xi)$$

$$\theta_{tot}(z) = \theta_0(z) + \varepsilon \theta_1(z)\,|_{z \to 0} = C$$

$$\theta_{tot}(z) = \theta_0(z) + \varepsilon \theta_1(z)\,|_{z \to \infty} = 0$$

$$u_{tot}(z) = u_0(z) + \varepsilon u_1(z)\,|_{z \to 0} = 0$$

$$u_{tot}(z) = u_0(z) + \varepsilon u_1(z)\,|_{z \to \infty} = 0$$

$$\theta_1(z = 0) = u_1(z = 0) = 0$$

$$\theta_1(z \to \infty) = u_1(z \to \infty) = 0$$

Final sol. for ε^1 part (1st order), while the whole solution = 0th + 1st order:

$$\theta_1(z) = \theta_A \exp(-\frac{z}{h_p})[-\frac{1}{15}\sin(\frac{z}{h_p}) - \frac{1}{6}\cos(\frac{z}{h_p})]$$
$$+\theta_A \exp(-\frac{2z}{h_p})[\frac{1}{15}\sin(\frac{2z}{h_p}) + \frac{1}{15}\cos(\frac{2z}{h_p}) + \frac{1}{10}]$$

$$\theta_A = \frac{h_p C^2 \mu \sin(\alpha)}{K},\; u_A = \frac{C^2\mu}{h_p \Gamma}$$

$$u_1(z) = u_A \exp(-\frac{z}{h_p})[-\frac{1}{3}\sin(\frac{z}{h_p}) + \frac{2}{15}\cos(\frac{z}{h_p})]$$
$$+u_A \exp(-\frac{2z}{h_p})[-\frac{1}{30}\cos(\frac{2z}{h_p}) + \frac{1}{30}\sin(\frac{2z}{h_p}) - \frac{1}{10}]$$