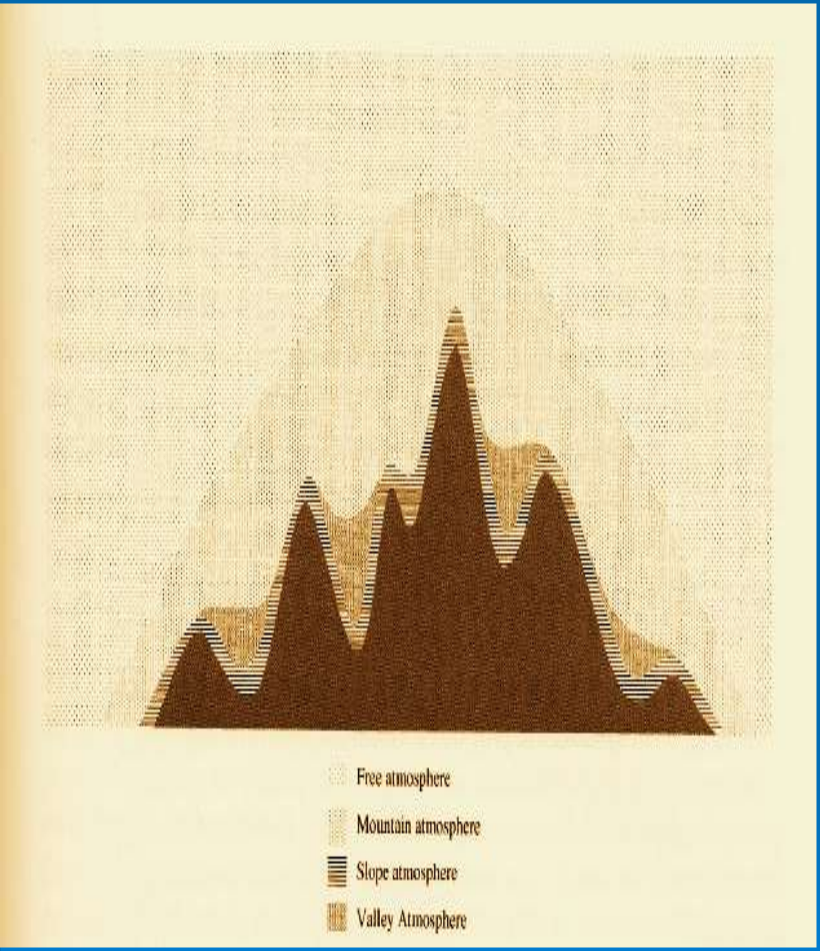
New developments in Prandtl model for slope flows

Branko Grisogono[1], Toni Jurlina[2], Ivan Gűttler[2] & Željko Večenaj[1]

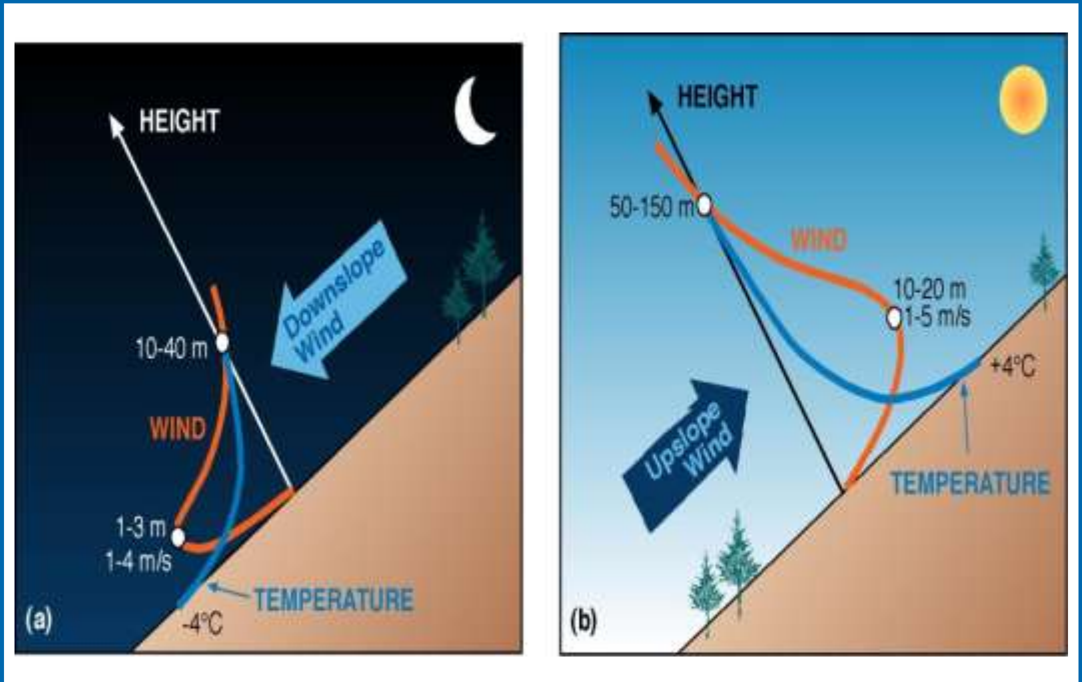
[1]Dept. of Geophysics, Faculty of Sci, Univ. of Zagreb
[2]Meteorological and Hydrological Service, Zagreb

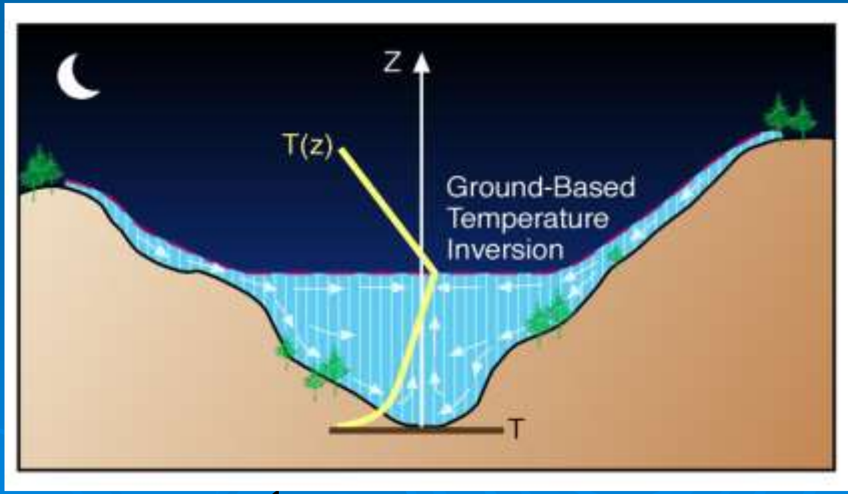
Workshop, Zagorje, Nov. 2014

…perspective…



After E. Ekhart 1948





After C.D. Whiteman 2000

CONTENT

INTRO

Motivation & Overview

PRANDTL MODEL

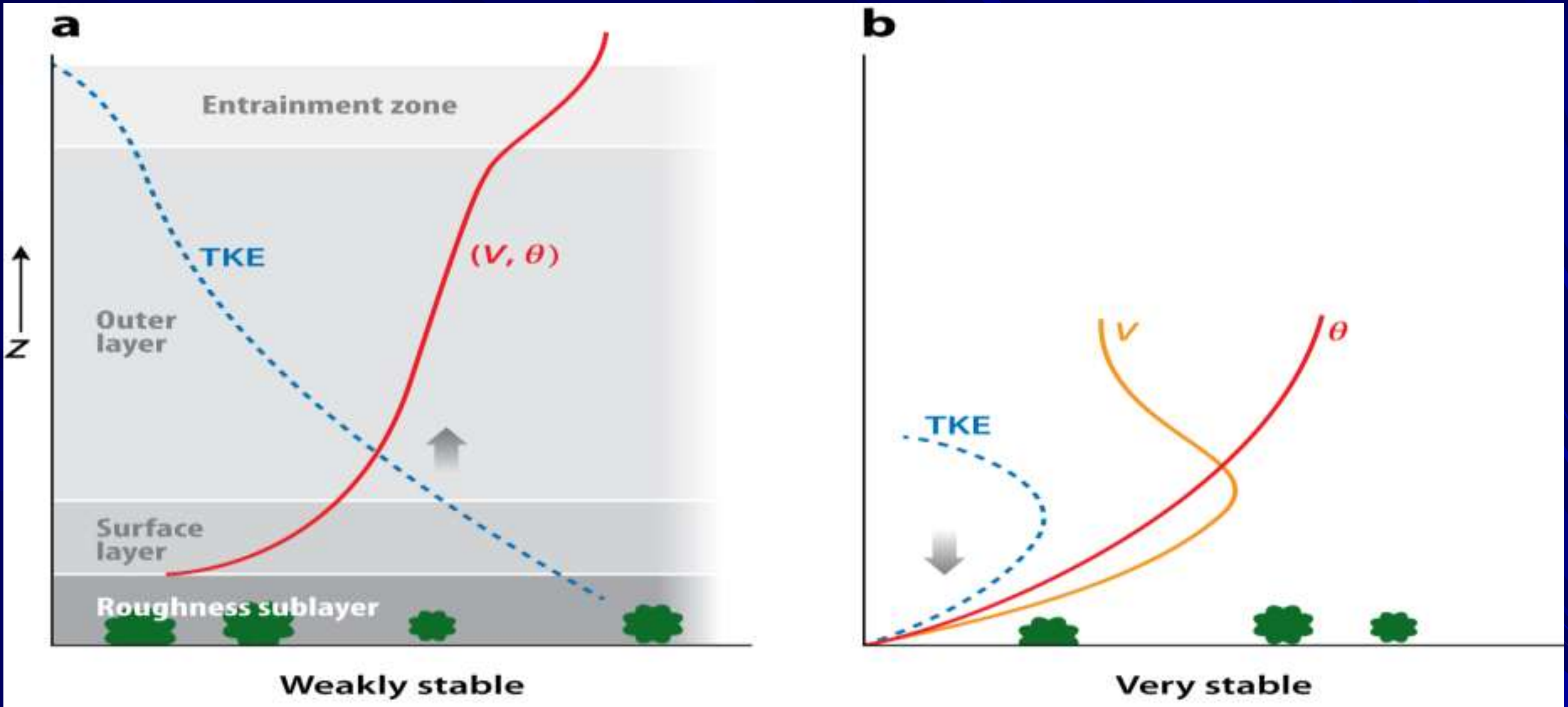
Weakly nonlinear, with $K(z)_{WKB}$ approximation, QJ2014

RESULTS & DISCUSSION

Energetics of the model

Further avenues …

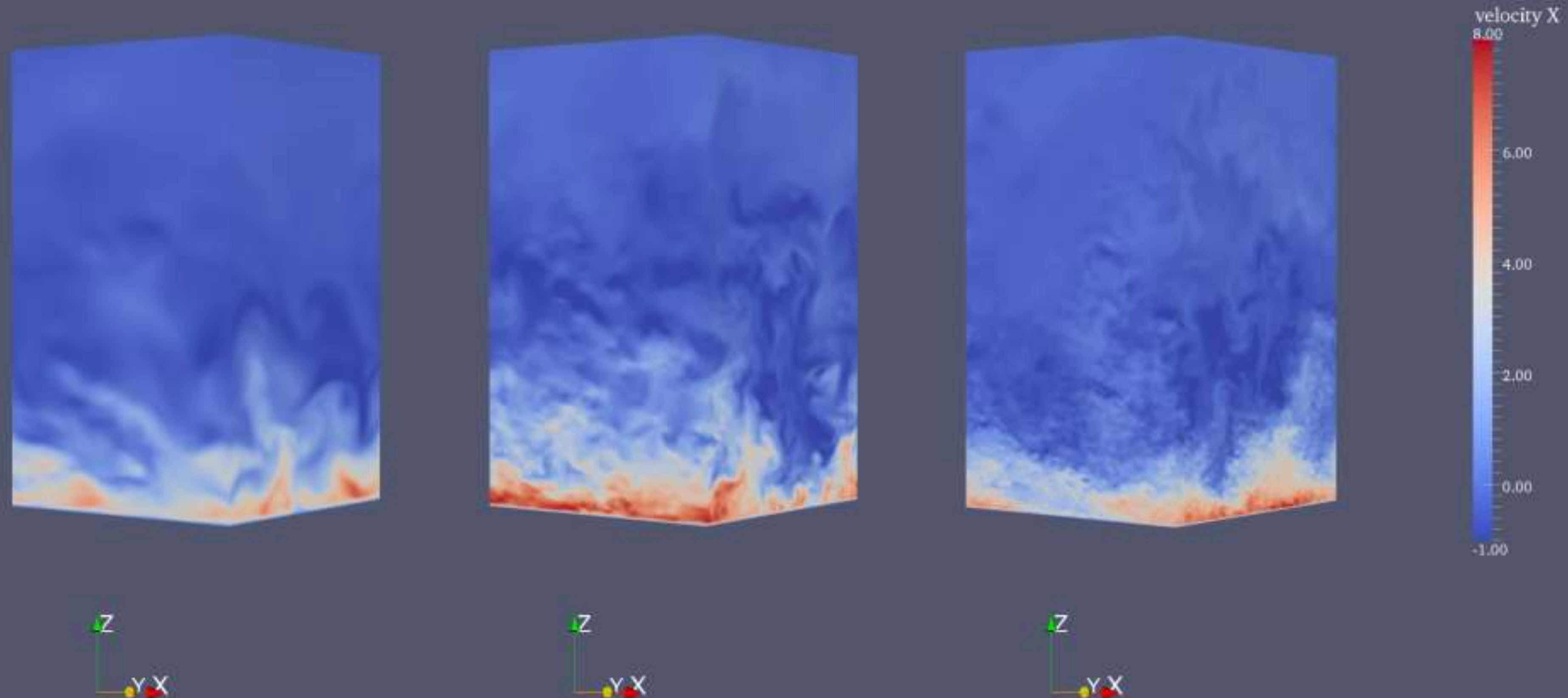
WSABL **VSABL**



Mahrt L. 2014.
Annu. Rev. Fluid Mech. 46:23–45

Climate models poorly treat diurnal mnt. cycle, coastal & mnt. flows, related precip
…Even NWP models often have those problems…
Is Total turbulent Energy, TE, relevant?

Marco Giometto, EPFL-EFLUM, DNS of katabatic flows, α = 60º, Re = 500, 10^3, 5000; left, mid-, right



CLASSIC PRANDTL MODEL

- *Stationarity for (θ , **U**) :* *t >* ***1-1.5T, T = 2π/(Nsin(α))***
- *PDE → ODE*
- ***(α, Γ, C) => θ(z), U(z)*** = *exp. - complex decay*

U:

$$0 = \frac{g}{\theta_0}\sin(\alpha)\theta + \frac{d}{dz}\left(\Pr K \frac{dU}{dz}\right)$$

θ:

$$0 = -\Gamma \sin(\alpha)U + \frac{d}{dz}\left(K \frac{d\theta}{dz}\right)$$

Modified: weakly-nonlinear Prandtl model

u:

$$0 = g\frac{\theta}{\Theta_0}sin(\alpha) + KPr\frac{\partial^2 u}{\partial z^2}$$

θ:

$$0 = -\left(\Gamma + \boldsymbol{\varepsilon}\frac{\boldsymbol{\partial\theta}}{\boldsymbol{\partial z}}\right)u\, sin(\alpha) + K\frac{\partial^2\theta}{\partial z^2}, \qquad 0 \leq \varepsilon \ll 1$$

- $u_{tot} = u_0 + \varepsilon u_1 + \varepsilon^2 u_2 + ..$
- $\theta_{tot} = \theta_0 + \varepsilon\theta_1 + \varepsilon^2\theta_2 + ..$

ε^1:

- $$\frac{\partial^4 u_1}{\partial z^4} + u_1 \frac{g\Gamma sin^2(\alpha)}{\Theta_0 Pr K^2} = -\frac{g sin^2(\alpha)}{\Theta_0 Pr K^2} u_0 \frac{\partial \theta_0}{\partial z}$$

- $$\frac{\partial^4 \theta_1}{\partial z^4} + \theta_1 \frac{g\Gamma sin^2(\alpha)}{\Theta_0 Pr K^2} = \frac{sin(\alpha)}{K} \frac{\partial^2}{\partial z^2}\left(u_0 \frac{\partial \theta_0}{\partial z}\right)$$

-Again: damped oscillator but now forced by the ε^0 state (RHS).

-Whole solution = 0^{TH} + 1^{ST} order ⬄ exp-decay with various coeff's … each (u_1, θ_1) has 5 terms…(Toni'J.'s diploma work)

WKB approximation - gradual K(z)

- *Solutions keep their original structure as with the const. coeff.*

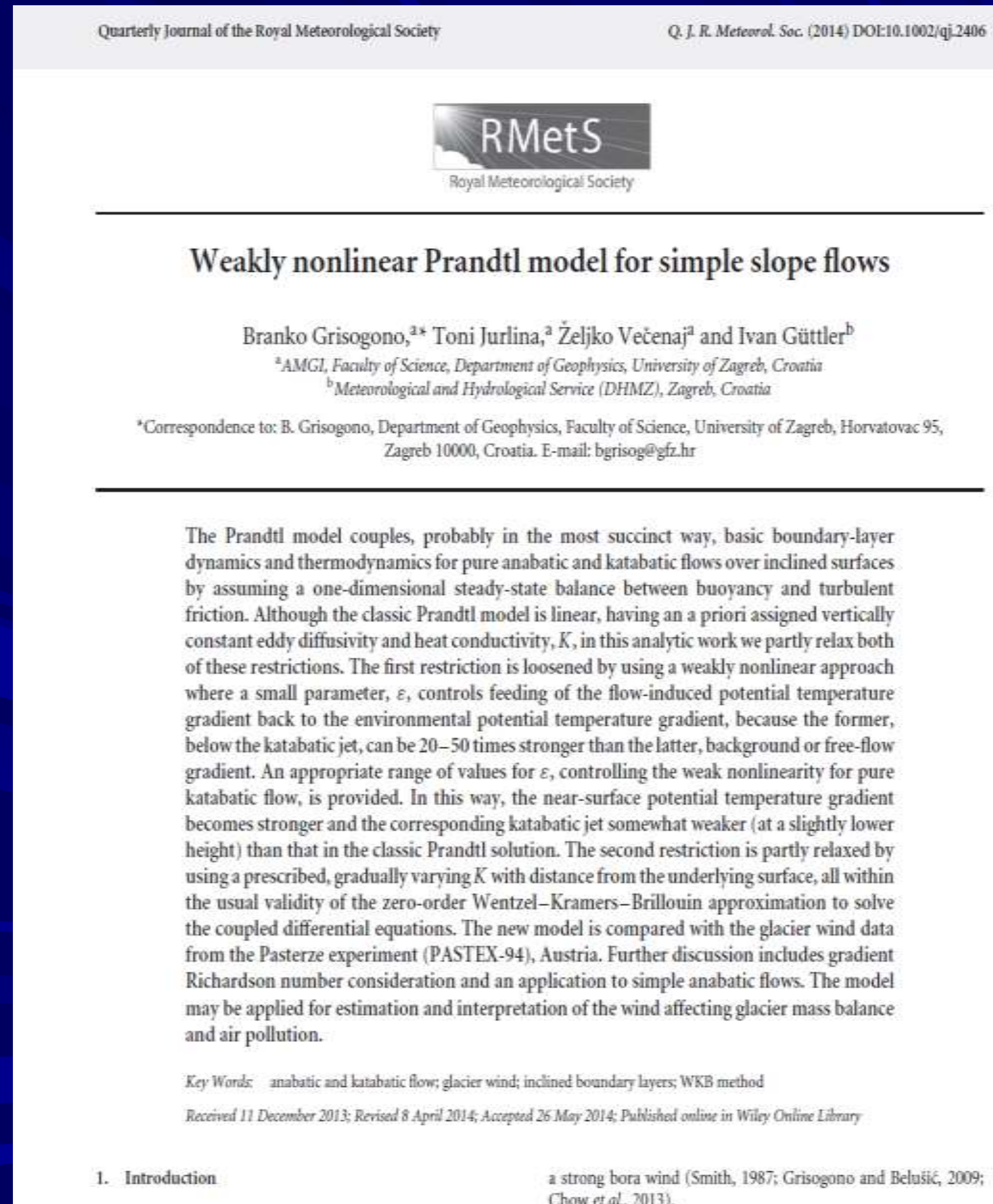
- *ODE solutions but now with local values, integrated from below if previously multiplied by z ⇔ 0^{TH} order WKB*

Chosen $K(z) = K_0\, z/h\, \exp\{-z^2/(2h^2)\}$, or similar, h > LLJ

details in JAS'01, QJ'01,… ACP'10, QJ'11,'14

- *The solution*
- *ε estimate*
- *K(z) values*
- *Data comparison*
- *Anabatic extension*
- *$Ri_{grad} \rightarrow \pm \infty$, etc.*

In QJ press
since July 2014



Quarterly Journal of the Royal Meteorological Society   *Q. J. R. Meteorol. Soc.* (2014) DOI:10.1002/qj.2406

RMetS
Royal Meteorological Society

Weakly nonlinear Prandtl model for simple slope flows

Branko Grisogono,[a*] Toni Jurlina,[a] Željko Večenaj[a] and Ivan Güttler[b]
[a]*AMGI, Faculty of Science, Department of Geophysics, University of Zagreb, Croatia*
[b]*Meteorological and Hydrological Service (DHMZ), Zagreb, Croatia*

*Correspondence to: B. Grisogono, Department of Geophysics, Faculty of Science, University of Zagreb, Horvatovac 95, Zagreb 10000, Croatia. E-mail: bgrisog@gfz.hr

The Prandtl model couples, probably in the most succinct way, basic boundary-layer dynamics and thermodynamics for pure anabatic and katabatic flows over inclined surfaces by assuming a one-dimensional steady-state balance between buoyancy and turbulent friction. Although the classic Prandtl model is linear, having an a priori assigned vertically constant eddy diffusivity and heat conductivity, K, in this analytic work we partly relax both of these restrictions. The first restriction is loosened by using a weakly nonlinear approach where a small parameter, ε, controls feeding of the flow-induced potential temperature gradient back to the environmental potential temperature gradient, because the former, below the katabatic jet, can be 20–50 times stronger than the latter, background or free-flow gradient. An appropriate range of values for ε, controlling the weak nonlinearity for pure katabatic flow, is provided. In this way, the near-surface potential temperature gradient becomes stronger and the corresponding katabatic jet somewhat weaker (at a slightly lower height) than that in the classic Prandtl solution. The second restriction is partly relaxed by using a prescribed, gradually varying K with distance from the underlying surface, all within the usual validity of the zero-order Wentzel–Kramers–Brillouin approximation to solve the coupled differential equations. The new model is compared with the glacier wind data from the Pasterze experiment (PASTEX-94), Austria. Further discussion includes gradient Richardson number consideration and an application to simple anabatic flows. The model may be applied for estimation and interpretation of the wind affecting glacier mass balance and air pollution.

Key Words: anabatic and katabatic flow; glacier wind; inclined boundary layers; WKB method

Received 11 December 2013; Revised 8 April 2014; Accepted 26 May 2014; Published online in Wiley Online Library

1. Introduction

a strong bora wind (Smith, 1987; Grisogono and Belušić, 2009; Chow *et al.*, 2013).

The Energetics of the nonlinear Prandtl model

$$\partial_t u = \frac{g}{\theta_0}\sin(\alpha)\theta + \frac{\partial}{\partial z}\left(\Pr K \frac{\partial u}{\partial z}\right) \mid \cdot u$$

$$\partial_t \theta = -u(\Gamma + \varepsilon \frac{\partial \theta}{\partial z})\sin(\alpha) + \frac{\partial}{\partial z}\left(K \frac{\partial \theta}{\partial z}\right) \mid \cdot a\theta$$

[a] = (m/s)^2 / K^2

= g/[$\theta_o \Gamma$]

$$\frac{\partial}{\partial t}\frac{(u^2 + a\theta^2)}{2} = \ldots = \partial_t (KE + PE) = \partial_t TE$$

$$\frac{\partial}{\partial t}\frac{(u^2 + a\theta^2)}{2} = K\left[\partial^2{}_{zz}\frac{P_r u^2 + a\theta^2}{2} - \left(P_r(\partial_z u)^2 + a(\partial_z \theta)^2\right)\right] - a\mathcal{E}\sin(\alpha)\left[u\theta\partial_z\theta\right]$$

∂_t (KE + PE) = ∂_t (TE) = turb. DIFF. - DISSipation + INTeraction

The forcing term, [g/θo] sin(α)·θ, appears to be only a conversion term, thus, gone in TE eqn.

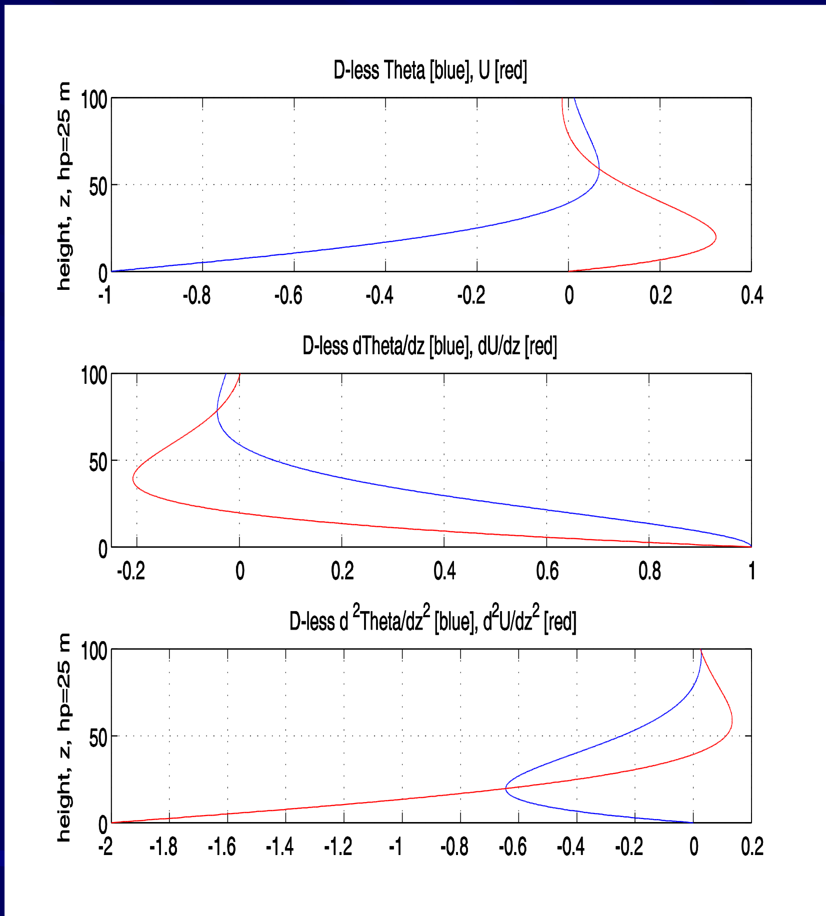
Idealized input ↓

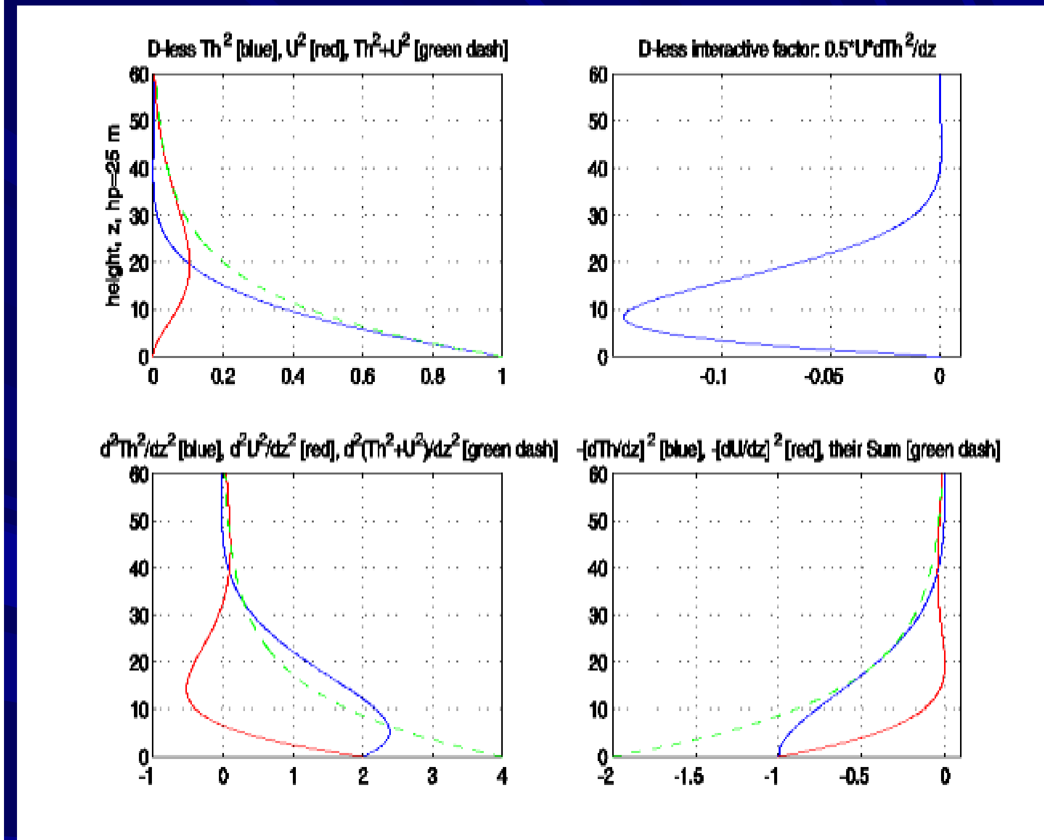
→TE profiles

INTeraction ↓

Larry M.'s TKE ↓ *red*

extreme at ≈π h_P/ 9



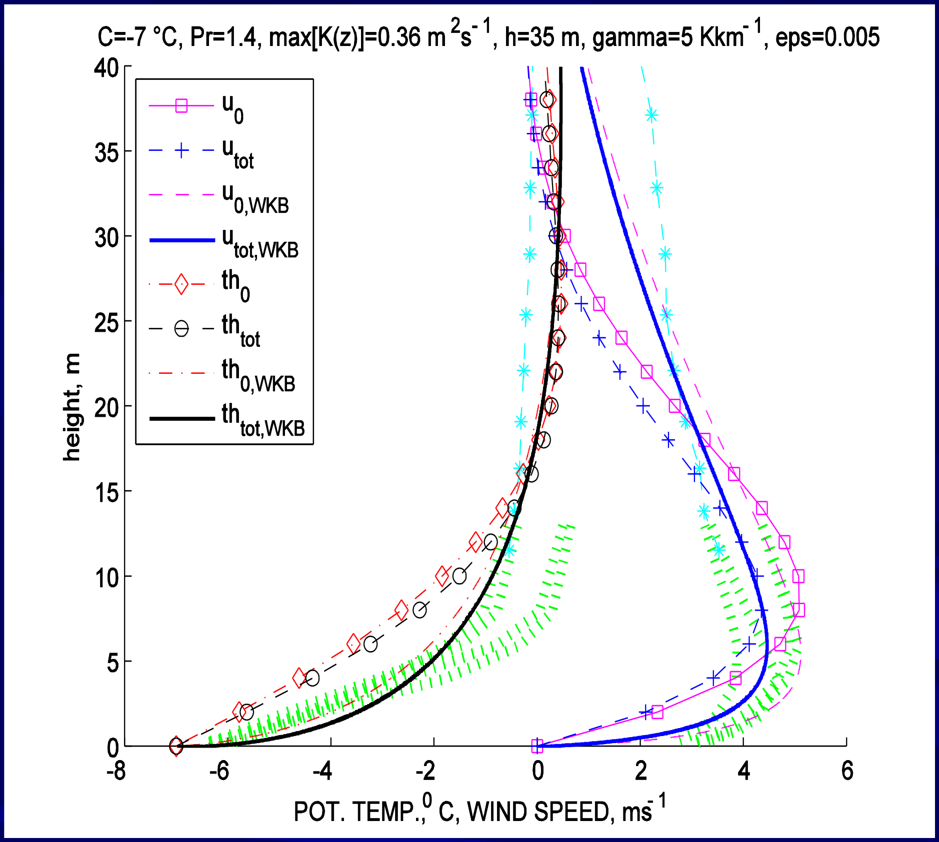


D-less DIFF ^{x} *2* ↑

DISS ↑

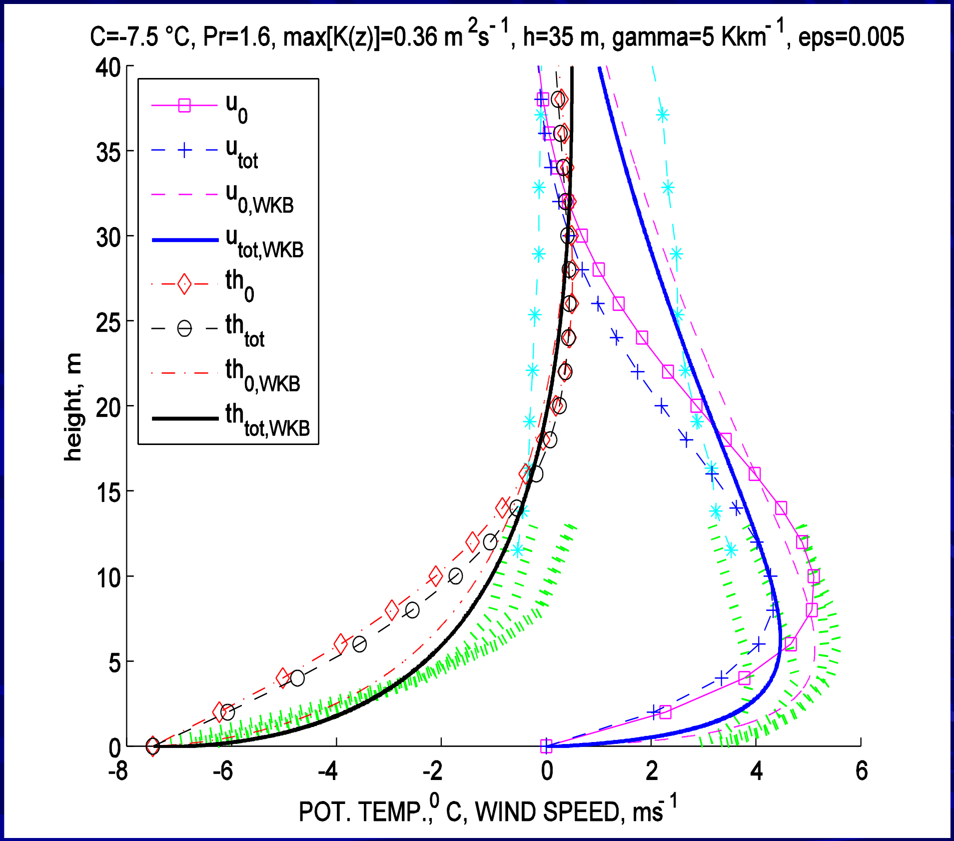
Note: as D-less plots made in the init. var's only, the higher derivatives grow artificially, though they scale at their level of order

PASTEX'94 data (8 level 13 m tower & balloon) vs. Prandtl model



tower = green - hatched

balloon = turquoise - star-dashed



Morning ↑ Afternoon →

Statistical comparison & the betterment given in QJ'14

Conclusions

Since Climate & NWP models treat diurnal mnt. cycle, coastal & mnt. flows, related precip…. poorly or inadequately…

Slope flows parameterization should be included in the models due to the models' inadequate [Δz, Δ_{HOR}] & parameterizations

Modified Prandtl model allows for stronger & sharper inv. during weak flows, large near-surface Ri_{grad} – missing in most of NWP & climate models; max LLJ @ $\pi h_P/4$ & max INT PE↔ KE @ $\pi h_P/9$

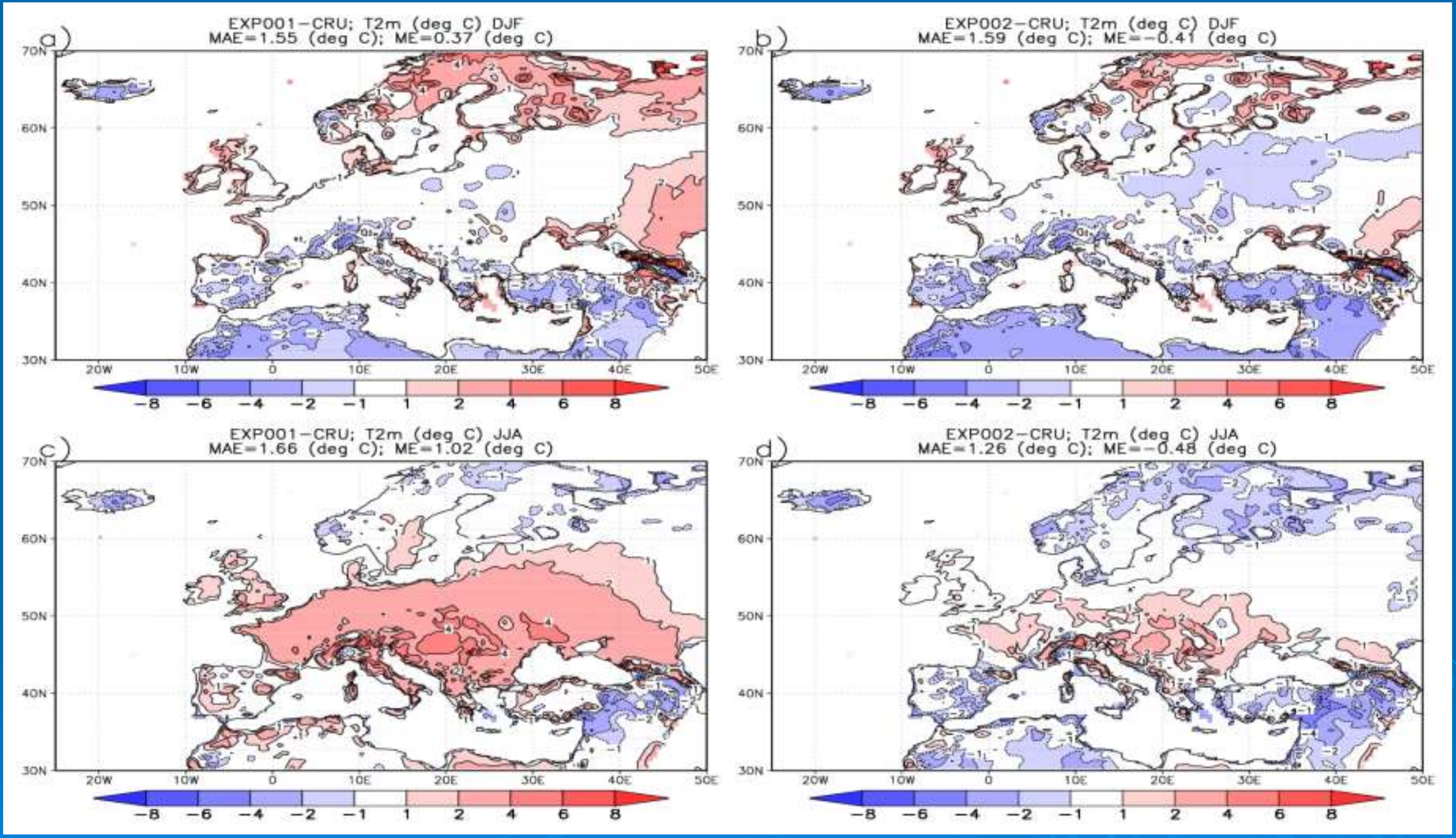
Total turbulent energy, TE, might be the proper concept to treat the VSABL & the LBC in the models

-Chances for diploma works, PhD ↔ parameterization ↔ postdocs, senior projects – climate modeling…

Spare slides

- Eqns!

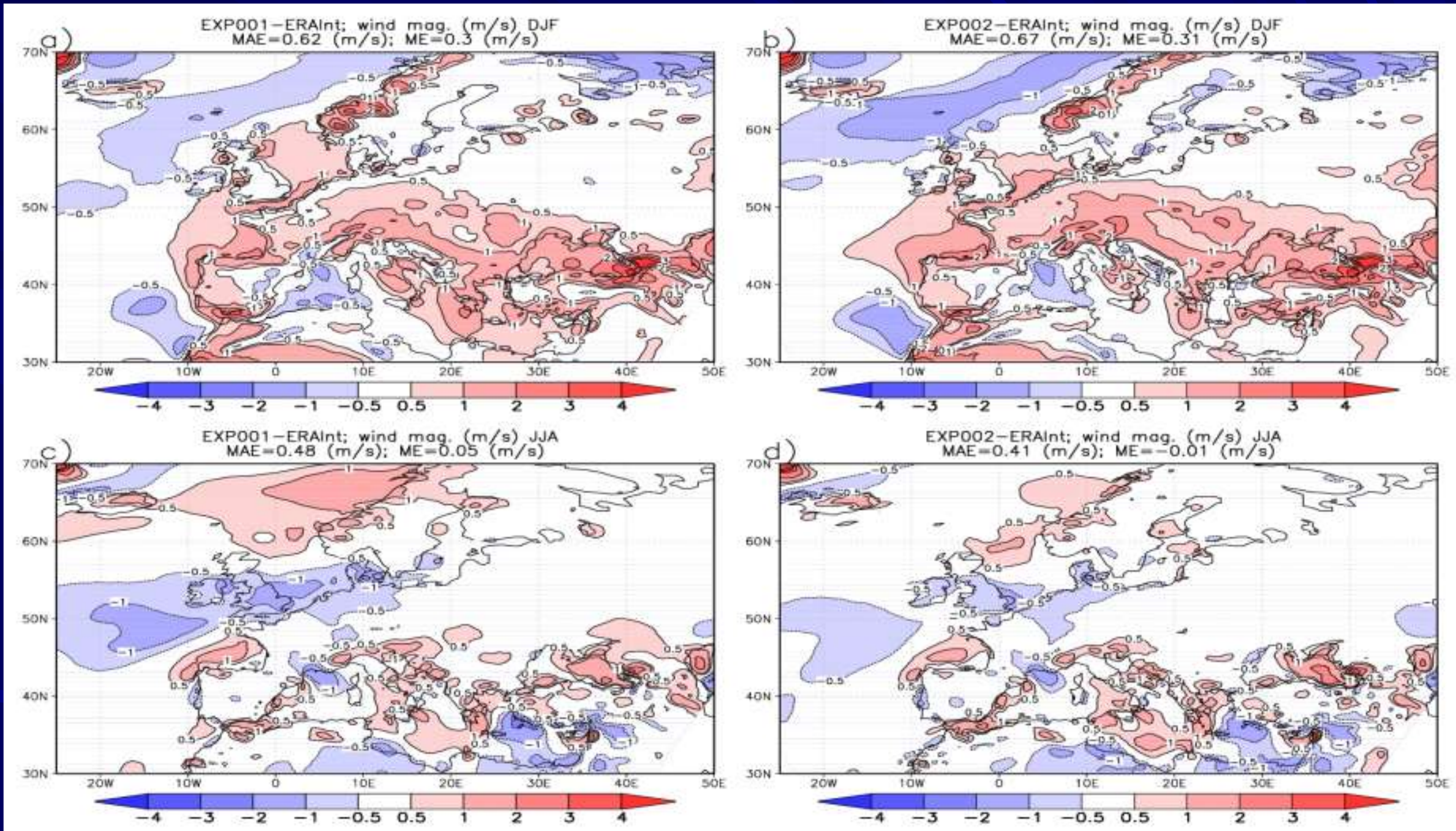
Typical reg. clim. simulation, RegCM, 1989-1998, dx ≈ 50 km



Winter & summer **T2m <u>"errors"</u>** ↑ 1[st] order turb. scheme

Same but with H.O.C. turbulence scheme ↑

Clim. ***simulations*** *&* ***obs*** *– cont'd - "errors" for wind simulations*



Winter & summer **wind magn.** errors, ↑ st order turb. scheme

Same but with H.O.C. turbulence scheme ↑

$$\frac{4}{h_p^4} = \frac{g\Gamma \sin^2(\alpha)}{\Theta_0 \Pr K^2}$$

$$h_p = \frac{\sqrt{2}}{\sigma}, \ \sigma = \sqrt{\frac{N \sin(\alpha)}{K \Pr^{1/2}}}, \ \mu = \sqrt{\frac{g}{\Theta_0 \Gamma \Pr}}, \ N^2 = \frac{\Gamma g}{\Theta_0}$$

$$\theta_{1,\mathrm{hom}} \sim \exp(\lambda\xi)$$

$$\xi = 2z/h_p$$

$$\theta_{1,\mathrm{hom}} = \mathrm{G}_1 \exp(-\frac{\xi}{2})\sin(\frac{\xi}{2}) + \mathrm{G}_2 \exp(-\frac{\xi}{2})\cos(\frac{\xi}{2})$$

$$\theta_{1,part} = A_1 \exp(-\xi)\cos\xi + B_1 \exp(-\xi)\sin\xi + C_1 \exp(-\xi)$$

$$\theta_{tot}(z) = \theta_0(z) + \varepsilon\theta_1(z)|_{z\to 0} = C$$

$$\theta_{tot}(z) = \theta_0(z) + \varepsilon\theta_1(z)|_{z\to\infty} = 0$$

$$u_{tot}(z) = u_0(z) + \varepsilon u_1(z)|_{z\to 0} = 0$$

$$u_{tot}(z) = u_0(z) + \varepsilon u_1(z)|_{z\to\infty} = 0$$

$$\theta_1(z=0) = u_1(z=0) = 0$$

$$\theta_1(z\to\infty) = u_1(z\to\infty) = 0$$

Final sol. for ε^1 part (1st order), while the whole solution = 0th + 1st order:

$$\theta_1(z) = \theta_A \exp(-\frac{z}{h_p})[-\frac{1}{15}\sin(\frac{z}{h_p}) - \frac{1}{6}\cos(\frac{z}{h_p})] + \theta_A \exp(-\frac{2z}{h_p})[\frac{1}{15}\sin(\frac{2z}{h_p}) + \frac{1}{15}\cos(\frac{2z}{h_p}) + \frac{1}{10}]$$

$$\theta_A = \frac{h_p C^2 \mu \sin(\alpha)}{K}, \ u_A = \frac{C^2\mu}{h_p\Gamma}$$

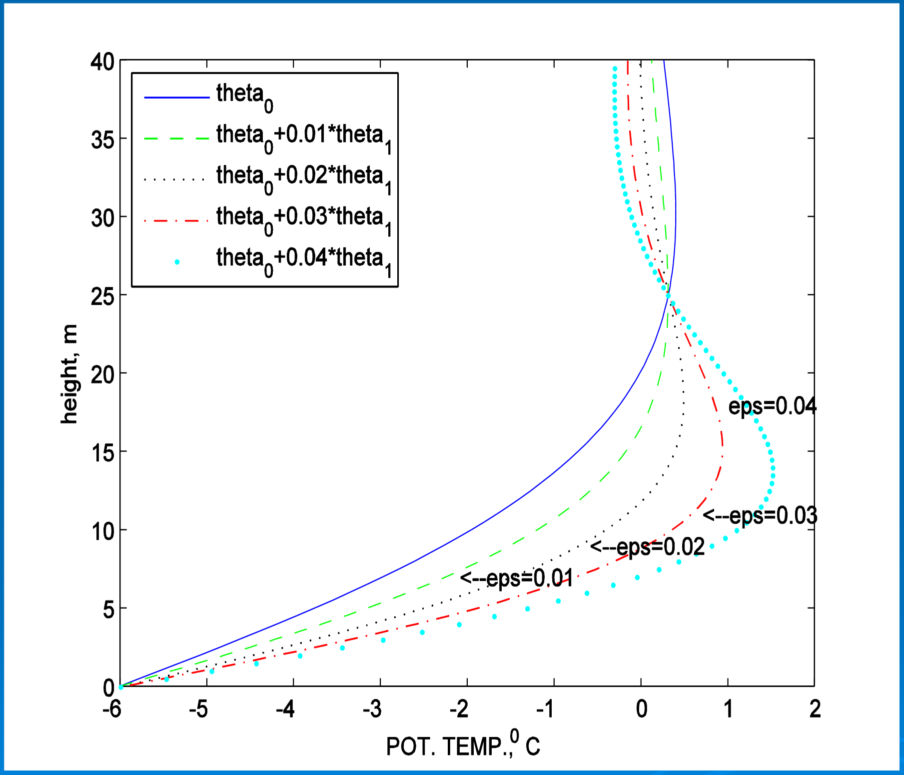
$$u_1(z) = u_A \exp(-\frac{z}{h_p})[-\frac{1}{3}\sin(\frac{z}{h_p}) + \frac{2}{15}\cos(\frac{z}{h_p})] + u_A \exp(-\frac{2z}{h_p})[-\frac{1}{30}\cos(\frac{2z}{h_p}) + \frac{1}{30}\sin(\frac{2z}{h_p}) - \frac{1}{10}]$$

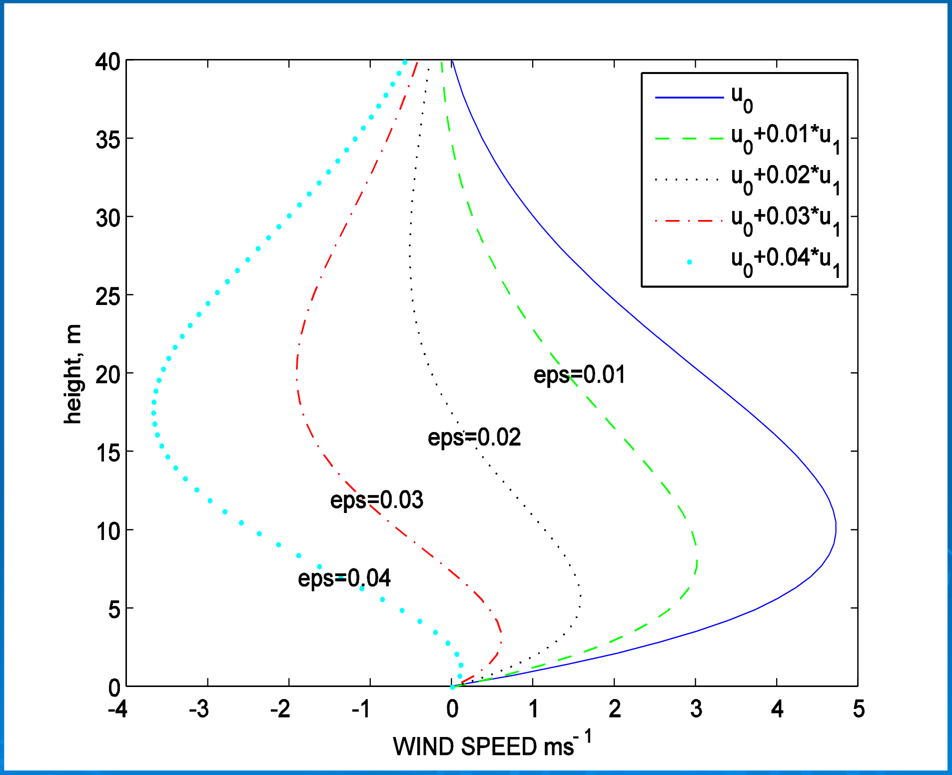
▪ *Estimating the small parameter ε, … expecting ε ≤ O(0.1)*

Thermodyn. Eqn →

$$\max(\varepsilon) \leq \frac{2\Gamma h_P}{|C|}$$

… Numerical experiments → max(ε) a bit smaller





here for Pasterze data: ε = 0.005 | *Anabatic flow: larger ε & K(z)*