Basis for the calculations of travel time curves in a 1D model

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Abstract

In this report we provide some basis of refraction and reflection *seismology,* which are used to describe the earth structure based on velocity models, to help in the understanding of simple model of the earth structure. We provide an algorithm using refraction *seismology*, to identify the velocities and calculate the depths of a multilayer model of the earth, using travel times data collected by *geophones* (seismograms) from artificial sources. This algorithm allows us to create a one dimensional model, which can be used as the input for calculating a 3D model of the earth structure through a seismic *tomography* aproach.

1 Introduction

The fundamental data for seismological studies of the earth's interior are the *travel times* of seismic waves, since they are used to learn about the velocity structure between the source and the receiver. Travel time curves depend on the characteristics of the media through which the seismic waves propagate[1]. Hence, the velocity's distribution with depth can be deduced from travel time curves. The first seismic waves ever used for the study of the Earth's structure were those produced by earthquakes, and remain, as of today, as the main source of information, especially for the study of the deep interior of the Earth. However, seismic waves generated by earthquakes have the limitation of the lack of control over the exact place and time of their origin. For this reason, another approach

[1]For a more detailed description of geometrical and physical details refer to the appendixes

was developed in which waves generated artificially by explosions and other methods are used. These studies are divided into two types: refraction and reflection, which uses waves refracted and reflected with large angles of incidence at long distances; and vertical reflection, which use waves reflected vertically at short distances. Our goal is to calculate velocities using arrival times to determine the earth structure, using mainly the refraction approach.

2 Refraction seismology approach

The measurements available on seismological studies of the earth's interior are the arrival times of the seismic waves at the receivers[2]. These instruments are a distances from the shot points that are large compared with the depth of the horizon to be mapped. To convert these to travel times, the origin and the location of the source must be known. Waves follow paths that depend on the velocity structure. For example, consider the travel time between two points. If the velocity v were constant, then the *ray path* p would be a straight line, and the velocity would be found by dividing the distance x between the two points by the travel time $v = \frac{x}{t}$. If, instead, an interface consist of separate media with different velocities, the *ray path* would consist of two straight lines with different slope, and the travel time would be the sum of the time spent along each segment. We show a strategy to address this problem in the following sections.

2.1 Methodology

This problem can be posed mathematically by writing the travel time t between the source a and the receivers b_i, where a and b_i are positive quantities that denote distance, as function of a and b by using the integral equation

$$t(z(a,b)) = t(z) = \int_a^{b_i} \frac{1}{v(z)}\, dz = \int_0^{b_i} s(z)\, dz \tag{1}$$

where $s = \frac{1}{v}$ is called the *slowness* and for simplicity let $a = 0$. The reason to work with the *slowness* instead of velocity is basically that the last integral in (1) is a linear function, which is better to use

[2] To illustrate the difference between travel time and arrival time let consider the following example: if two earthquakes occurred at the same place but exactly 24 hours apart, the wave travel times would be the same but the arrival times would differ by one day.

for the discretization of the problem. Thus we have a continous *inverse problem* here, where our data are the travel times t and the model is the *slowness* s. For problems where an artificial source like an explosion or a sledge hammer is used, we are interested only in studying the *P-waves.* The travel times give an integral constraint on the velocity distribution, but do not indicate which of the many paths satisfying this constraint were followed by rays. Thus we need a big set of travel times to provide enough information to get an adequate distribution of velocity.

The simplest approach to this *inverse problem*[3] is to treat the earth as a set of flat layers of uniform-velocity material. The method is based on the analysis of travel times and amplitudes of recorded waves for layered media of constant velocities and continuous distributions of velocity with respect to the depth. We refer the reader to appendix A for more mathematical details on the derivation of the formulas for the different types of waves. There are three basic ray paths (Figure 1) from a source on the surface at the origin to a surface receiver at distance x.

1. The first ray path corresponds to a *direct wave* D that travels through the top layer.
2. The second ray path is for a wave reflected from the interface between the top layer and the immediate lower layer. Because the angles of incidence and reflection are equal, the wave reflects halfway between the source and the receiver. This curve is a hyperbola, as seen in Figure 2. At distances much greater than the layer thickness, the travel time for the reflected wave approaches asymptotically that of the direct wave.
3. The third type of wave is the head wave, often referred to as a refracted wave. This wave results when a down-going wave impinges on the interface equal to or greater than the critical angle [4].

[3] Once a model is chosen to represent the earth, seismological data (travel time) is used to estimate the parameters of the model.

[4] Critical angle is the least angle of incidence at which total internal reflection takes place, the angle of incidence is measured with respect to the normal at the refractive boundary. The critical angle is $\theta_c = \sin^{-1}\left(\frac{v_0}{v_1}\right)$

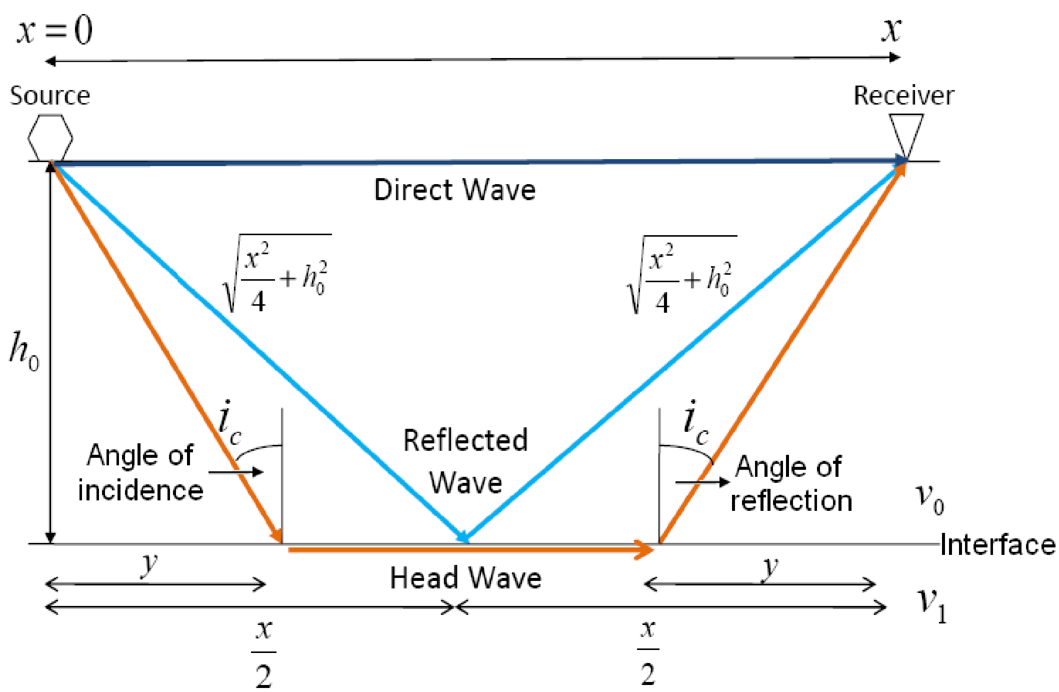

Figure 1: Travel time curves(Refraction *seismology*)

The simple structure in Figure 1 is described for the following parameters:

- The velocities v of each layer and the *halfspace* between the layers. Here we use the notation v_i to indicate the velocity at the i_{th} layer .
- The *layer thickness* h_i of each layer, that can be calculated first by using the *crossover* distance x_d, and later the intercept of each head wave.[5]

According to the refraction *seismology* approach, velocities increase with depth, therefore $\frac{1}{v_0} > \frac{1}{v_1}$, the direct wave's travel time curve has a higher slope but starts at the origin, whereas the head wave has a lower slope but a nonzero intercept (Figure 2). The direct waves are the first arrival for the receivers that are closer to the source than the cross over distance x_d. Beyond this point, the head waves arrive first. The *inverse problem* of finding the velocity structure at various depths can be solved from the travel times observed at the surface as a function of the source-receiver distance.

[5]The *crossover* distance is the distance where the head wave is the first arrival wave to the receivers.

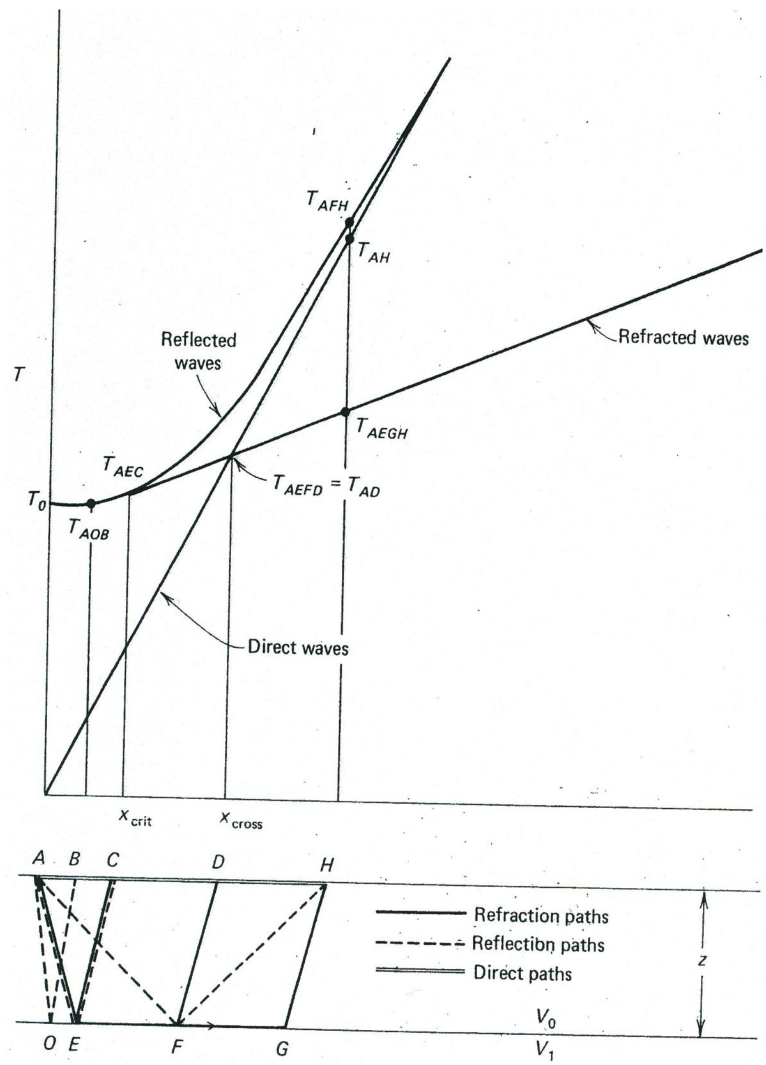

Figure 2: Relation between times for waves reflected and refracted from a horizontal interface [Dobrin].

The forms of travel time curves, critical distances of reflected waves, and slopes of refracted waves, together with distributions of amplitudes, allow the determination of velocity models for the Earth's *crust.* Figure 2 shows the form of the refraction waves as identified from a seismogram like the ones in Figure 3.

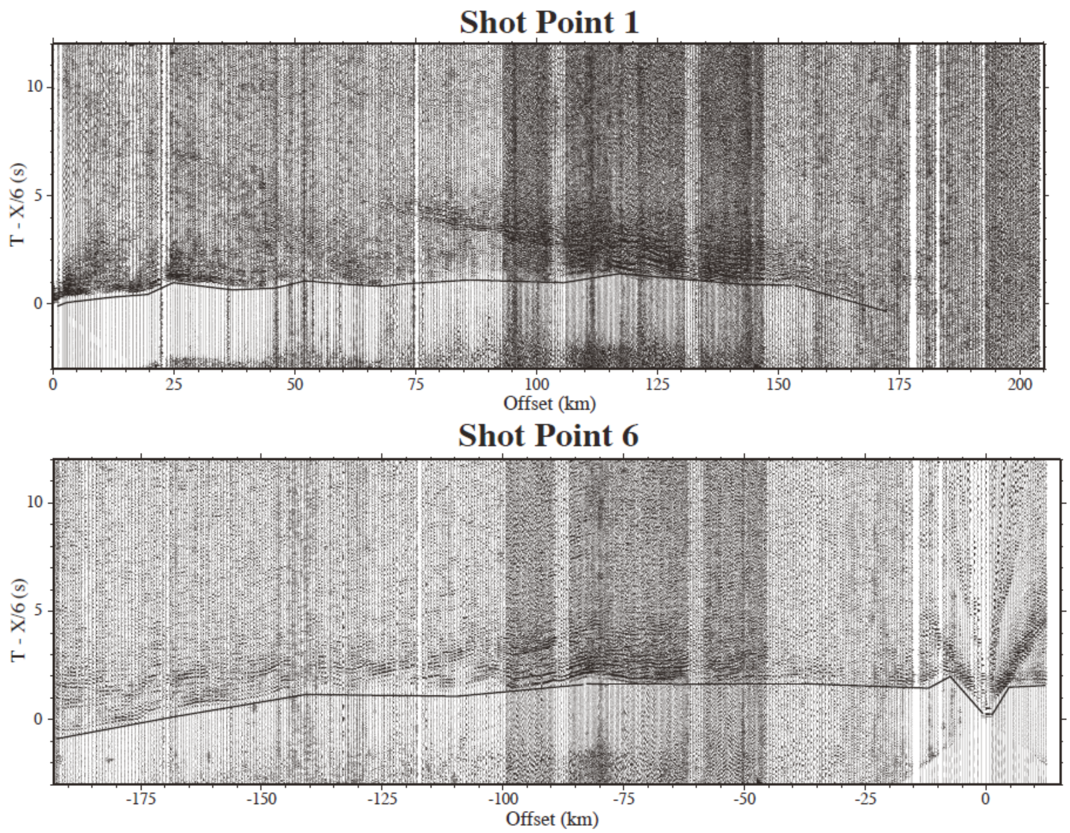

Figure 3: Samples of two seismograms where the refracted waves can be drawn.

Figure 4 depicts an experiment using seven shot points and n receivers among the shot points.

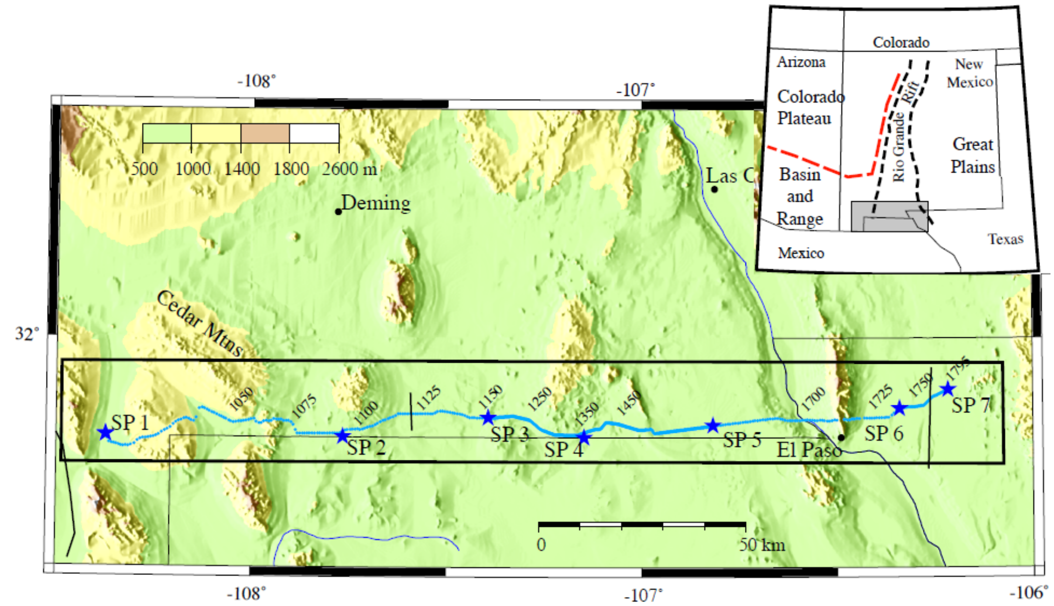

Figure 4: Shot points and receivers

2.2 Implementation

As mentioned before, the refraction model is the simplest approach to the inverse problem. The standard procedure for this model begins by plotting the times obtained from the experimentation at each receiver (geophone) against distance. Let's recall that this approach bases its analyzes on the first arrival times, hence we are interested in the first point plotted at each of the receivers. The next step is to identify straight lines in our plot, or in other words, slopes and intercepts. By calculating the equation of the lines we are now able to find the slopes (hence the velocities) of the first arrival times and the intercepts with the y axis. Moreover, by setting the equations of the first two lines equal to each other we can find the crossover distance (see appendix B for mathematical formulations). Note that once the velocities of the first two layers and the crossover distance are known, it is now possible to find the thickness of the first layer. This procedure is applied iteratively for all the layers.

If we have n receivers spaced at the same distance and m layers, to solve the inverse problem we can proceed with the following algorithm:An algorithm to calculate the layer thickness for a model with m layers, once the travel times are picked using geophones and velocities are calculated from seismograms is shown below as Algorithm 1.

Algorithm 1 1D refraction model

Step1 Plot data obtained from the *geophones*

Step2 Find the slope of the travel curves $T_D(x) = \frac{1}{v_0}x$, $T_{H_j}(x) = \frac{1}{v_j}x + \tau_j$ and their intercepts $\tau_j = T_{H_j}(0)$, for $j = 2, .., m$. Here $\tau_1 = 2h_0\left(\frac{1}{v_0^2} - \frac{1}{v_1^2}\right)^{1/2}$

Step3 Identify the *crossover* distance x_d by solving $T_D(x) = T_H(x)$.

Step4 Find the layer thickness h_0 by solving

$$x_d = 2h_0\left(\frac{v_1 + v_0}{v_1 - v_0}\right)^{1/2}$$

and then (if $v_j < v_{j+1}$, $\forall j > 1$) using the following iterative formula for the other layers

$$h_{m-1} = \frac{\tau_m - 2\sum\limits_{j=0}^{m-2} h_j\left(\frac{1}{v_j^2} - \frac{1}{v_m^2}\right)^{1/2}}{2\left(\frac{1}{v_{m-1}^2} - \frac{1}{v_m^2}\right)^{1/2}} \tag{2}$$

Notice that for the second layer $m = 2$ and thickness= h_1, since the first layer $m = 1$ and thickness= h_0, is given by the formula 2 as $h_1 = \frac{\tau_2 - 2\sum\limits_{j=0}^{0} h_j\left(\frac{1}{v_j^2} - \frac{1}{v_2^2}\right)^{1/2}}{2\left(\frac{1}{v_{2-1}^2} - \frac{1}{v_2^2}\right)^{1/2}} = \frac{\tau_2 - 2h_0\left(\frac{1}{v_0^2} - \frac{1}{v_2^2}\right)^{1/2}}{2\left(\frac{1}{v_1^2} - \frac{1}{v_2^2}\right)^{1/2}}$. The simple geometries used give models that fit data reasonably well and provide starting models for more sophisticated analyses, that can approximate more the complex reality. Although the refraction method does not give as much information as precise a structural picture as reflection, it provides data on velocity of the refracted beds which often allow the geophysicist to identify them or to specify their lithology. The method usually makes it possible to cover a given area in a shorter time and more economically than with the reflection method.

3 Reflection seismology approach

Instead of studying the refraction arrivals (Figure 2), the most widely used geophysical techniques uses the *reflection arrivals* (Figure 5) to determine velocities within the *crust*, by measuring the time required for a seismic wave or pulse generated in the earth by near surface artificial (controlled) seismic source of energy, such as dynamite, a specialized air gun or vibrators, to return to the surface after reflection form interface between formation having different physical properties. Reflection data are densely sampled in space and time (multiple sources - multiple receivers), then it has greater resolution than the one obtained using refraction methodology. Variations in the reflection times from place to place on surface usually indicate structural features in the strata below. Reflection data have also been used for identifying lithology, generally from velocity and attenuation characteristics, and for detecting hydrocarbons, primarily gas, directly on the basis of reflection amplitudes and other seismic indicators.

The method makes it possible to produce structural maps of any geological horizons which may yield reflections, but the horizons themselves cannot be identified without independent geologic information such as might be obtained from wells. With reflection methods one can locate and map such features as anticline, faults, salt domes, and reefs.

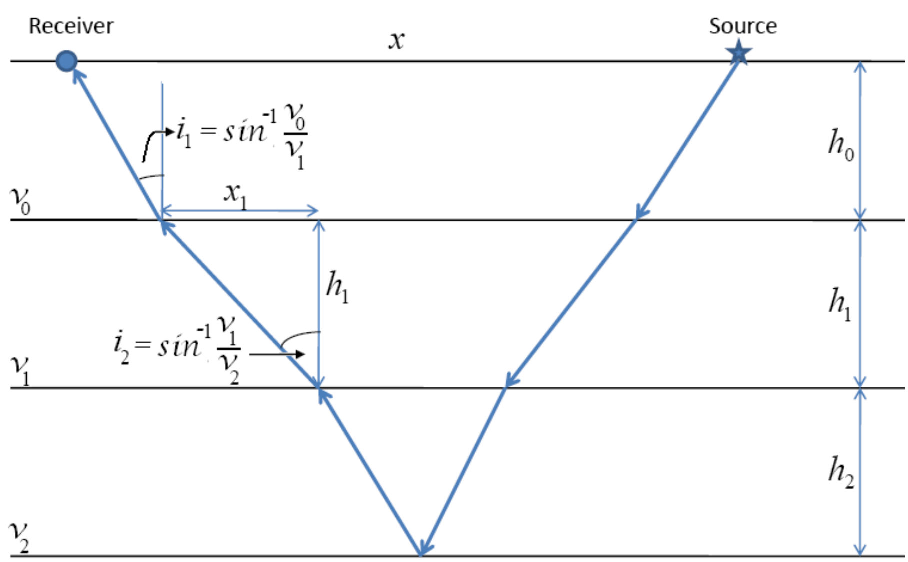

Figure 5: Travel time curves for reflection *seismology*

3.1 Travel time curves for reflection

We first consider the simplest geometry: one flat layer with uniform-velocity material. We know already from refraction *seismology* that for a layer of thickness h_0 with velocity v_0 has travel time

$$T(x)^2 = \frac{x^2}{v_0^2} + 4\frac{h_0^2}{v_0^2} = \frac{x^2}{v_0^2} + \underbrace{\left(2\frac{h_0}{v_0}\right)^2}_{t_0} = \frac{x^2}{v_0^2} + t_0^2 \tag{3}$$

as a function of x, which is the distance between the source and the receiver.

This computation is known as the *offset* in reflection *seismology*, and it is represented by a hyperbola. It is plotted for reflection *seismology* with time increasing downward, because later arrivals reflect deeper in the earth. Now we use another kind of formulation for data analysis. Note that ΔT_j the one-way travel time in the j^{th} layer with velocity v_j, is related to the thickness, h_j, and the horizontal distance traveled, $x = x_j$ (Figure 2), by

$$v_j \Delta T_j = (x_j^2 + h_j^2)^{1/2} \tag{4}$$

It easy to see that

$$\sin i_j = \frac{x_j}{(x_j^2 + h_j^2)^{1/2}} = \frac{x_j}{v_j \Delta T_j} \quad \text{and} \quad \cos i_j = \frac{h_j}{(x_j^2 + h_j^2)^{1/2}} = \frac{h_j}{v_j \Delta T_j}$$

equation (4) can be written as:

$$v_j \Delta T_j \quad = \quad \frac{x_j^2 + h_j^2}{(x_j^2 + h_j^2)^{1/2}} = x_j \sin i_j + h_j \cos i_j \qquad \text{or}$$

$$\Delta T_j = p_j x_j + \eta_j h_j \tag{5}$$

where

$$p_j = u_j \sin i_j \qquad \text{and} \qquad \eta_j = u_j \cos i_j$$

The ray parameter p_j, or horizontal *slowness*, and η_j or vertical *slowness* are the components of the *slowness* vector

$$u_j^2 = p_j^2 + \eta_j^2 \tag{6}$$

which points in the direction of the wave propagation. From (5) the total travel is the sum over all layers, with a factor of two(downgoing and upgoing legs), this is

$$T(x) = 2\sum_{j=0}^{n} \Delta T_j = 2\sum_{j=0}^{n} p_j x_j + 2\sum_{j=0}^{n} \eta_j h_j$$

By Snell's law the horizontal ray parameter is constant along the ray path, thus

$$T(x) = px + 2\sum_{j=0}^{n} \eta_j h_j \tag{7}$$

where x is the total horizontal distance traveled. Now lets define the *intercept-slowness* function, using the equation (6) as

$$\tau(p) = 2\sum_{j=0}^{n} \eta_j h_j = 2\sum_{j=0}^{n} (u_j^2 - p_j^2)^{\frac{1}{2}} h_j \tag{8}$$

then the total travel time can be given by

$$T(x) = xp + \tau(p) \tag{9}$$

and then we are ready to formulate an algorithm, shown below as Algorithm 2, to calculate the velocities and the thickness of each layer using the intercept *slowness* formulation [Stein].

Algorithm 2 1D reflection model(ray parameter)

Step1 Find the slope(*total slowness*) $u_j^2 = \frac{1}{v_j^2}$, and the intercept τ_j of the travel time curve (3) (Here T^2 is function of x^2)

$$T(x)^2 = \frac{x^2}{v_j^2} + t_j^2 \quad \left(T_H(x) = \frac{x}{v_j} + \tau_j \right)$$

Step2 Find the ray parameter p for each layer j solving equation (9). ($T(x) = xp + \tau_j(p)$)

Step3 Solve (6) to find the vertical *slowness* η_j for each layer j.

Step4 Use the discrete formula (7) to find the thickness h_j of each layer j.

3.2 Refraction vs reflection

The principle difference between the geometry of the refraction and that of the reflection is in the interaction that takes place between the seismic waves and the lithological boundaries they encounter in the course of their propagation. The waves reflected by the boundaries travel along paths which are quite easy to visualize. Refracted waves of the type used in exploration work follow a somewhat more complicated trajectory that may not be as obvious to one's institution. Refraction paths cross boundaries between materials having different velocities in such a way that energy travel from source to receiver in shortest possible time. Most refraction prospecting involves the use of waves having trajectories along the tops of layers with speed that are appreciably greater than those of any overlaying formations. The speeds and depths of such layers are determined from the times required for refracted waves to travel between source and receivers which are also in the surface. The distances between the two are almost always several times as great as the depths of the boundaries which the waves travel.

The wave-path geometry associated with refraction prospecting requires considerably different source-geophone geometry in the field than is used for reflection surveys. In mapping structures at any particular depth, the shot and geophones must be farther apart for refraction than they would be for reflection. In most refraction work only the initial arrival of seismic energy is recorded, although later arrivals are sometimes used if conditions are favorable. Because of the greater distance traveled the frequency of refraction signals tends to lower than that of reflections. The recording requirements are thus different, and the instruments employed are likely to have different characteristics.

Finally, in spite of the numerous advantages, refraction employed much less extensively than reflection in oil exploration. This is probably attributable to the greater amount of explosives required, the larger scale of field operations, and lower precision in the structural information obtained from this method.

Some final remarks:

- Because the slope decreases with increasing velocity, "flatter" travel time curves indicate higher velocities.
- The relation between the travel time curve and ray paths, is given by the following equation

  $$p = \frac{\sin i_j}{v_j}$$

  where i is the incidence angle in the $j_{th}-$layer. And because of Snell's law, the ray parameter p is constant along the ray. Thus

  $$p = \frac{\sin i_j}{v_j} = \frac{\sin i_0}{v_0}$$

  where i_0 is the incidence angle of the top layer.

Appendix A

Geometrical Optics

Law of Reflection

When a ray of light is reflected at an *interface* dividing two optical media, the reflected ray remains within the *plane of incidence*, and the angle of reflection θ_t equals the angle of incidence θ_i(Figure 6). The *plane of incidence* is the plane containing the incident ray and the surface normal at the point of incidence.

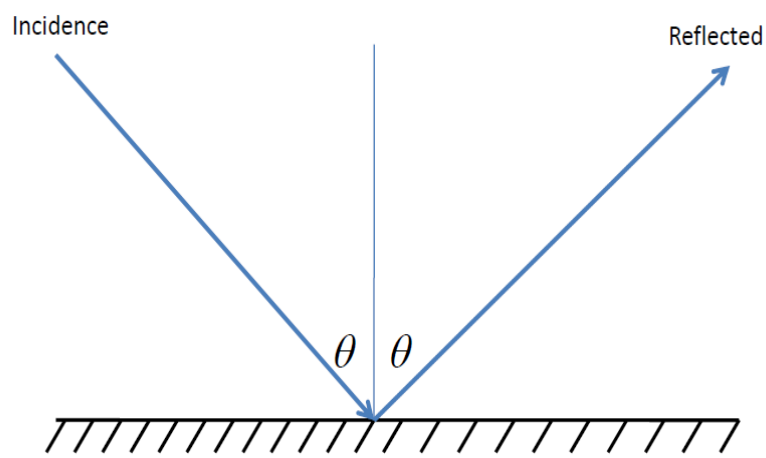

Figure 6: Reflection law

Law of Refraction (Snell's Law)

When a ray of light is refracted at an interface dividing two transparent media, the transmitted ray remains within the plane of incidence and the sine of the angle of refraction θ_t is directly proportional to the sine of the angle of incidence θ_i. (Figure 7)

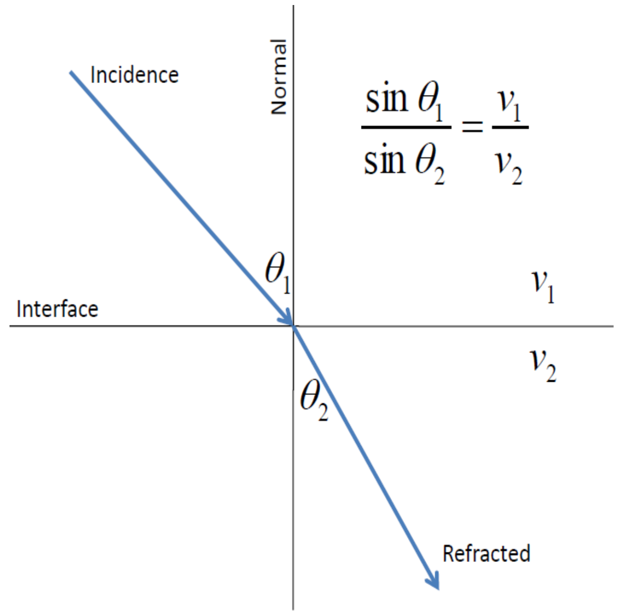

Figure 7: Snell's law

Critical Angle

If the angle of refraction θ_2 is equal to 90°, then the refracted wave travels horizontally, giving us as a result a head wave. When this is the case, from Snell's law we obtain the following relation:

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} \Rightarrow \frac{\sin\theta_1}{\sin 90°} = \frac{v_1}{v_2} \Rightarrow \sin\theta_1 = \frac{v_1}{v_2}$$

The angle θ_1 that generates a refracted angle θ_2 equal to 90° is called the *critical angle*.

Huygen's principle

Huygen's principle (Figure 8) recognizes that each point of an advancing wave front is in fact the center of a fresh disturbance and the source of a new train of waves; and that the advancing wave as a whole may be regarded as the sum of all the secondary waves arising from points in the medium already traversed.

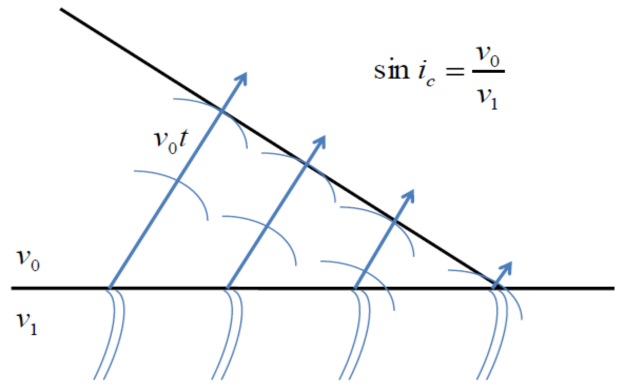

Figure 8: Huygen's principle

Appendix B

Mathematical Details

Flat Layer Method

In this section we describe with some mathematical detail the derivation of the formulas for each one of the refracted waves showed in the section 2 (Figure 1), based on the Snell's Law.

$$\text{Snell's Law: } \frac{\sin\theta_1}{v_1} = \frac{\sin\theta_0}{v_0}. \text{ If } sin\theta_1 = 90°, \text{ then } sin\theta_1 = \frac{v_0}{v_1}.$$

$$\tan(i_c) = \frac{y}{h_0} \Rightarrow y = h_0 \tan(i_c) \qquad h_0 = \text{Layer Thickness} \qquad v_0, v_1 = \text{Velocities}$$

Direct Wave

A direct wave travels directly through the layer, and its time is given by the following equation:

$$v = \frac{x}{t} \Rightarrow t = \frac{x}{v}$$

$$T_D(x) = \frac{x}{v_0}$$

If we look carefully at the previous equation, we note that the slope is $\frac{1}{v_0}$, and going through the origin since the x intercept is at $x = 0$.

Reflected Wave

The reflected wave reflects halfway between the source and the receiver, and its time is given by:

$$v = \frac{x}{t} \Rightarrow T_R(x) = \frac{x}{v} = \frac{\sqrt{\frac{x^2}{4} + h^2} + \sqrt{\frac{x^2}{4} + h^2}}{v_0}$$

$$T_R(x) = \frac{2\sqrt{\frac{x^2}{4} + h_0^2}}{v_0}$$

We can check by rewriting the previous equation that this is in fact the equation of a hyperbola with its y-intercept given by:

$$T_R(0) = \frac{2\sqrt{\frac{(0)^2}{4}+h_0^2}}{v_0} = \frac{2\sqrt{h_0^2}}{v_0} = \frac{2h_0}{v_0}$$

This reflected wave approaches asymptotically the direct wave.

Refracted Wave (Head Wave)

A head wave impinges on the interface at an angle at or beyond the critical angle. The wave travels down to the interface, then travels just below the interface with the velocity of the lower medium, and finally leaves the interface at the critical angle and travels upward to the surface.

By the concept of critical angle, we have $sin\theta = \frac{v_0}{v_1}$. Also, by using trigonometric identities we obtain the following:

$$\begin{aligned} cos\theta &= \sqrt{1-sin^2\theta} = \sqrt{1-\frac{v_0^2}{v_1^2}} \Rightarrow \frac{1}{cos\theta} = \sqrt{\frac{v_1^2}{v_1^2-v_0^2}} \\ v = \frac{x}{t} &\Rightarrow T_H(x) = \frac{x}{v} = \frac{\frac{h_0}{cos(i_c)}}{v_0} + \frac{x-2(h_0tan(i_c))}{v_1} + \frac{\frac{h_0}{cos(i_c)}}{v_0} \end{aligned}$$

$$\begin{aligned} T_H(x) &= \frac{2h_0}{v_0cos(i_c)} + \frac{x-2(h_0tan(i_c))}{v_1} = \frac{x}{v_1} + 2h_0\left[\frac{1}{v_0cos(i_c)} - \frac{tan(i_c)}{v_1}\right] \\ &= \frac{x}{v_1} + 2h_0\left[\frac{1}{v_0cos(i_c)} - \frac{\frac{sin(i_c)}{cos(i_c)}}{v_1}\right] = \frac{x}{v_1} + 2h_0\left[\frac{1}{v_0cos(i_c)} - \frac{sin(i_c)}{v_1cos(i_c)}\right] \\ &= \frac{x}{v_1} + 2h_0\left[\frac{1}{cos(i_c)}\left(\frac{1}{v_0} - \frac{sin(i_c)}{v_1}\right)\right] = \frac{x}{v_1} + 2h_0\left[\frac{1}{cos(i_c)}\left(\frac{1}{v_0} - \frac{\frac{v_0}{v_1}}{v_1}\right)\right] \\ &= \frac{x}{v_1} + 2h_0\left[\frac{1}{cos(i_c)}\left(\frac{1}{v_0} - \frac{v_0}{v_1^2}\right)\right] = \frac{x}{v_1} + 2h_0\left[\frac{1}{cos(i_c)}\left(\frac{v_1^2-v_0^2}{v_0v_1^2}\right)\right] \\ &= \frac{x}{v_1} + 2h_0\left[\sqrt{\frac{v_1^2}{v_1^2-v_0^2}}\left(\frac{v_1^2-v_0^2}{v_0v_1^2}\right)\right] = \frac{x}{v_1} + 2h_0\left[\sqrt{\frac{v_1^2(v_1^2-v_0^2)}{(v_1^2-v_0^2)v_0^2v_1^4}}\right] \\ &= \frac{x}{v_1} + 2h_0\sqrt{\frac{1}{v_0^2v_1^2}} = \frac{x}{v_1} + 2h_0\left[\frac{1}{v_0^2} - \frac{1}{v_1^2}\right] \\ T_H(x) &= \frac{x}{v_1} + 2h_0\sqrt{\frac{1}{v_0^2} - \frac{1}{v_1^2}} \end{aligned}$$

If we look carefully at the previous equation, we note that the slope is $\frac{1}{v_1}$, and when $x = 0$ it intercepts the y-axis at

$$T_H(x) = 2h_0\sqrt{\frac{1}{v_0^2} - \frac{1}{v_1^2}}$$

As we can note from the formulations, the direct wave's travel time curve has a higher slope but starts at the origin, whereas the head wave has a lower slope but it does not start at the origin. This means that at first, the direct wave's travel times are less than those of the head waves (the reflected waves are never the ones with the less travel times). However, after the critical distance, the head wave's travel times are less than those of the direct waves, even though it travels a larger distance. The *crossover distance* is found by looking at the interception between the direct and head waves, meaning setting both equations equal to each other.

$$\begin{aligned}
T_D(x) &= T_H(x) \\
\frac{x}{v_0} &= \frac{x}{v_1} + 2h_0\sqrt{\frac{1}{v_0^2} - \frac{1}{v_1^2}} \\
x[\frac{1}{v_0} - \frac{1}{v_1}] &= 2h_0\sqrt{\frac{1}{v_0^2} - \frac{1}{v_1^2}} \\
x &= 2h_0\frac{\sqrt{\frac{1}{v_0^2} - \frac{1}{v_1^2}}}{\frac{1}{v_0} - \frac{1}{v_1}} = 2h_0\frac{\sqrt{\frac{1}{v_0^2} - \frac{1}{v_1^2}}}{\sqrt{\frac{(v_1-v_0)^2}{v_0^2 v_1^2}}} \\
&= 2h_0\sqrt{\frac{(v_1^2 - v_0^2)(v_0^2 v_1^2)}{(v_1 - v_0)^2(v_0^2 v_1^2)}} = 2h_0\sqrt{\frac{(v_1^2 - v_0^2)}{(v_1 - v_0)^2}} \\
&= 2h_0\sqrt{\frac{(v_1 - v_0)(v_1 + v_0)}{(v_1 - v_0)^2}} \\
x &= 2h_0\sqrt{\frac{v_1 + v_0}{v_1 - v_0}}
\end{aligned}$$

Appendix C

Glossary

Algorithm. An algorithm is a sequence of well-defined instructions for completing a task, often used for calculation and data processing.

Crossover-Distance. Distance at which the direct and head wave travel time curves cross. . Beyond this point the head wave is the first arrival.

Crust. Outermost solid shell of the Earth. Ranges from 5 to 70 km in depth.

Geophone. Device that converts ground movement into voltage, which may be recorded. The deviation of this measured voltage from the baseline is called the seismic response and is analyzed for structure of the Earth.

Halfspace. Last layer considered for the tomography analysis.

Inverse-Problem. An inverse problem is the task that often occurs in many branches of science and mathematics where the values of some model parameter(s) must be obtained from the observed data.

Mantle. Second layer of the Earth, below the crust. Earth's mantle extends to a depth of 2890 km, making it the thickest layer of the Earth.

Mohorovicic-Discontinuity. Boundary between the crust and mantle, defined by a contrast in seismic velocity.

P-wave. Pressure or primary waves, are longitudinal waves that travel at maximum velocity within solids and the therefore the first waves to appear in a seismogram.

Ray-Path. Imaginary path along which travels the energy associated with a point on a wavefront.

Reflection. [See Appendix A]

Refraction. [See Appendix A]

S-wave. Shear or secondary waves, are transverse waves that travel more slowly the P-waves and thus appear later in a seismogram. Particle motion is perpendicular to the direction of wave propagation.

Seismology. Seismology is the scientific study of earthquakes and the propagation of elastic waves through the Earth.

Slowness. In seismology, slowness s is defined as the inverse of velocity.

$$velocity = \frac{distance}{time} \Rightarrow \frac{1}{velocity} = \frac{time}{distance} = s$$

Travel,times Travel time is a relative time, it is the number of minutes, seconds, etc. that the wave took to complete its journey.

Arrival,time The arrival time is the time when we record the arrival of a wave - it is an absolute time, usually referenced to Universal Coordinated Time (a 24-hour time system used in many sciences).

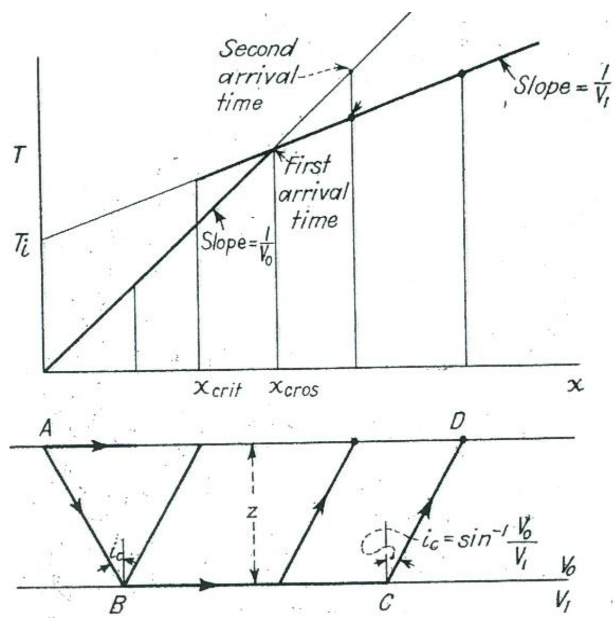

Figure 9: Ray paths of least time and time-distance curve for a layer over a horizontal interface [Dobrin].

Tomography. Tomography is imaging by sections or sectioning. A device used in tomography is called a tomograph, while the image produced is a tomogram. The method is used in medicine, archeology, biology, geophysics, oceanography, materials science, astrophysics, and other sciences. In most cases it is based on the mathematical procedure called tomographic reconstruction.

Wave,Direct. [See Appendix B]

Wave,Head. [See Appendix B]

Wave,Reflected. [See Appendix B]

Wave,Refracted. [See Appendix B]

References

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