27) Nađite brzinu rasprostiranja Rayleighevih valova u sredstvu u kojem se longitudinalni šire brzinom 5,36 km/s, a transverzalni 3,55 km/s!

$\beta = 3,55$ km/s
$\alpha = 5,36$ km/s

$c_R = ?$

tražimo korijene Rayleigheve jednadžbe:

$$f(\xi) = \xi^3 - 8\xi^2 + 8\xi\left(3 - 2\frac{\beta^2}{\alpha^2}\right) - 16\left(1 - \frac{\beta^2}{\alpha^2}\right) = 0, \quad \xi = \frac{c_R^2}{\beta^2}$$

$$f(\xi) = \xi^3 - 8\xi^2 + 17\xi - 9 = 0$$

→ Mora se znati riješiti kubna jednadžba !!

$f(0) = -9$
$f(1) = 1$
$0 < \xi < 1$

Fizikalni smisao ima samo ono rješenje koje zadovoljava $0 < c_R < \beta$ odnosno $0 < \xi < 1$

HORNER

	1	-8	17	-9
0,8	1	-7.2	11.2	0,00

$\Rightarrow \xi = 0.8$

$c_R^2 = \xi \cdot \beta^2 = 10.08 \frac{km^2}{s^{-2}}$

$\Rightarrow c_R = 3,2$ km s^{-1}

28)

Koja je brzina rasprostiranja R. valova c_R u postocima od β za sredstvo u kojem je vrijednost Poissonova omjera ν jednaka 0.2?

$$\nu = 0{,}2 = \frac{\lambda}{2(\lambda+\mu)}$$

$$\lambda = 0.4(\lambda+\mu)$$

$$\lambda - 0.4\lambda = 0.4\mu$$

$$0{,}6\lambda = 0.4\mu$$

$$\lambda = 0.67\mu$$

$$\frac{\beta^2}{\alpha^2} = \frac{\mu}{\lambda+2\mu} = \frac{1}{2{,}67} = 0.37$$

$$f(\xi) = \xi^3 - 8\xi^2 + 8\xi\left(3 - 2\frac{\beta^2}{\alpha^2}\right) - 16\left(1 - \frac{\beta^2}{\alpha^2}\right) = 0,$$

$$\xi = \frac{c_R^2}{\beta^2}$$

$$\xi^3 - 8\xi^2 + 8\xi(2{,}26) - 16(0{,}63) = 0$$

$$\xi^3 - 8\xi^2 + 18{,}08\xi - 10.08 = 0$$

	1	-8	18.08	-10.08
0,833	1	-7.167	12.11	ϕ

$\Rightarrow$ $\xi = 0.833$

$$c_R^2 = \xi b^2$$

$$c_R = \beta\sqrt{\xi} \Rightarrow \boxed{c_R = 0{,}91\beta}$$

Za Loveove valove u sloju iznad poluprostora izvedite (cut off) KRITIČNE frekvencije za prvi i drugi mod.
Sredstvo je sljedećih karakteristika:

$\beta_1 = 3.9 \ km s^{-1}$

$\beta_2 = 4.6 \ km s^{-1}$

$\rho_1 = 2.8 \ g cm^{-3}$

$\rho_2 = 3.3 \ g cm^{-3}$

$H = 40 \ km$

$\omega_{cn} = ?$

$k = \frac{2\pi}{\lambda}$

$f = \frac{\omega}{2\pi}$

$c = \frac{\omega}{k}$

$T = \frac{\lambda}{c}$

Izraz za periodsku jednadžbu za Loveove valove (tj. jedn. disperzije):

$$tg\left[\omega H \sqrt{\frac{1}{\beta_1^2} - \frac{1}{c_L^2}}\right] = \frac{\mu_2 \sqrt{\frac{1}{c_L^2} - \frac{1}{\beta_2^2}}}{\mu_1 \sqrt{\frac{1}{\beta_1^2} - \frac{1}{c_L^2}}}$$

implicitno izražava vezu između fazne brzine c valnog broja k odnosno frekvencije ω

$\beta_1 < c_L < \beta_2$

$$\xi = \frac{H}{c_L}\sqrt{\frac{c_L^2}{\beta_1^2} - 1}$$

Brzina ne ovisi samo o fizikalnim i geom. svojstvima sredstva (μ, β, H) već i o valnoj duljini $\lambda = \frac{2\pi}{k}$

odnosno periodu $T = \frac{\lambda}{c}$

Valovi različitih valnih duljina rasprostiru se različitim brzinama: DISPERZIJA

To je periodička f-cija čija se rješenja znaju do na π (±nπ) ⇒ općenito će postojati više rješenja koja zovemo modovima.

$\xi = 0 \Rightarrow c_L = \beta_1$

$\xi = max \Rightarrow c_L = \beta_2$

$$tg(\omega\xi) = \frac{\mu_2}{\mu_1} \frac{\sqrt{\frac{1}{c_L^2} - \frac{1}{\beta_2^2}} \cdot H}{\xi}$$

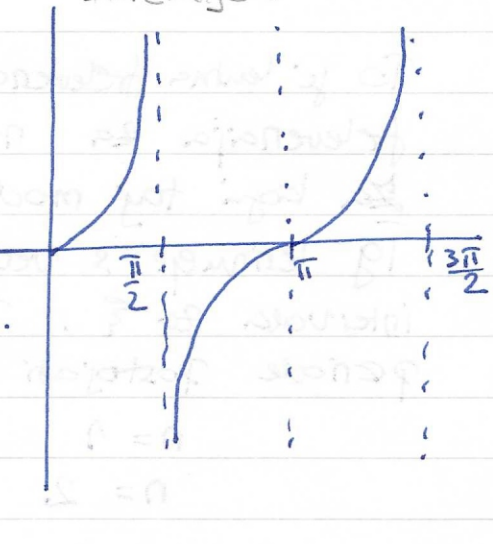
lijeva strana

Nul točke:

$tg(\omega\xi) = 0 \Rightarrow \omega\xi = n\pi \Rightarrow \xi = \frac{n\pi}{\omega}$

$tg(\omega\xi) = \infty \Rightarrow \omega\xi = \frac{(2n+1)\pi}{2} \Rightarrow \xi = \frac{\pi}{2\omega}, \frac{3\pi}{2\omega}, \ldots$

Presecanje jednadžbe DISPERZIJE:



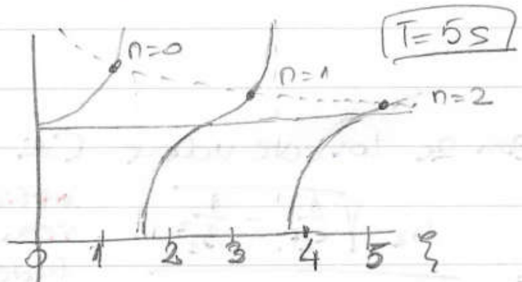
Razmotrimo što će se dogoditi za velike periode ili male frekvencije: ⇒ za nule tg krivuje:

$$\xi = \frac{n\pi}{\omega} \Rightarrow \text{raste}$$

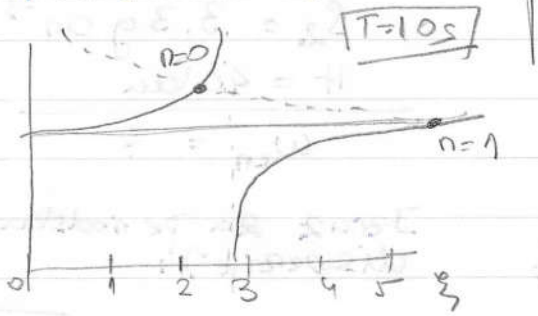
Razmak između tg krivulja $= \frac{\pi}{\omega}$ raste

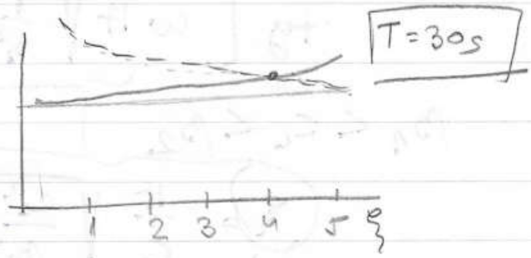
Kao rezultat toga → sve je manje tg krivulja za koje je $\frac{n\pi}{\omega} < \xi_{max}$

DISPERZIJA JEDNADŽBE DISPERZIJE ZA LOVE-ove valove u dvoslojnom poluprostoru



$\beta_1 = 3.9$ km s^{-1}
$\beta_2 = 4.6$ km s^{-1}
$\rho_1 = 2.8$ g cm^{-3}
$\rho_2 = 3.3$ g cm^{-3}
$H = 40$ km





Za svaku danu kutnu frekvenciju rješenje s najvećom mogućom vrijednosti ξ se dešava kad je: $\xi_{max} \Rightarrow c_L \equiv \beta_2$

Tada je $\quad tg\, \omega \xi_{max} = 0 \Rightarrow \omega \xi_{max} = n\pi$

$$\Rightarrow \quad \omega = \omega_{cn} = \frac{n\pi}{H\sqrt{\frac{1}{\beta_1^2} - \frac{1}{\beta_2^2}}} = \frac{n\pi}{5.44}$$

To je kutna frekvencija koja se naziva KRITIČNA (cutoff) kutna frekvencija za n-ti mod i to je najniža frekvencija za koji taj mod postoji

Tg krivulje s većim vrijednostima n su izvan dozvoljenog intervala za ξ. Dakle, za zadovoljavajuće duge periode postojati će samo osnovni mod.

$n = 1 \qquad \omega_{c1} = 0.58$ Hz $\quad \leftarrow \left(\omega_1 = \frac{\pi}{5.44}\right)$

$n = 2 \qquad \omega_{c2} = 1{,}16$ Hz $\quad \leftarrow \left(\omega_2 = \frac{2\pi}{5.44}\right)$

Love wave dispersion:

$$\tan\left[(\omega h/c_x)(c_x^2 / \beta_1^2 - 1)^{1/2}\right] = \frac{\mu_2(1 - c_x^2/\beta_2^2)^{1/2}}{\mu_1(c_x^2/\beta_1^2 - 1)^{1/2}}$$

Because the square roots are real (for a real solution),

$\beta_1 < c_x < \beta_2$

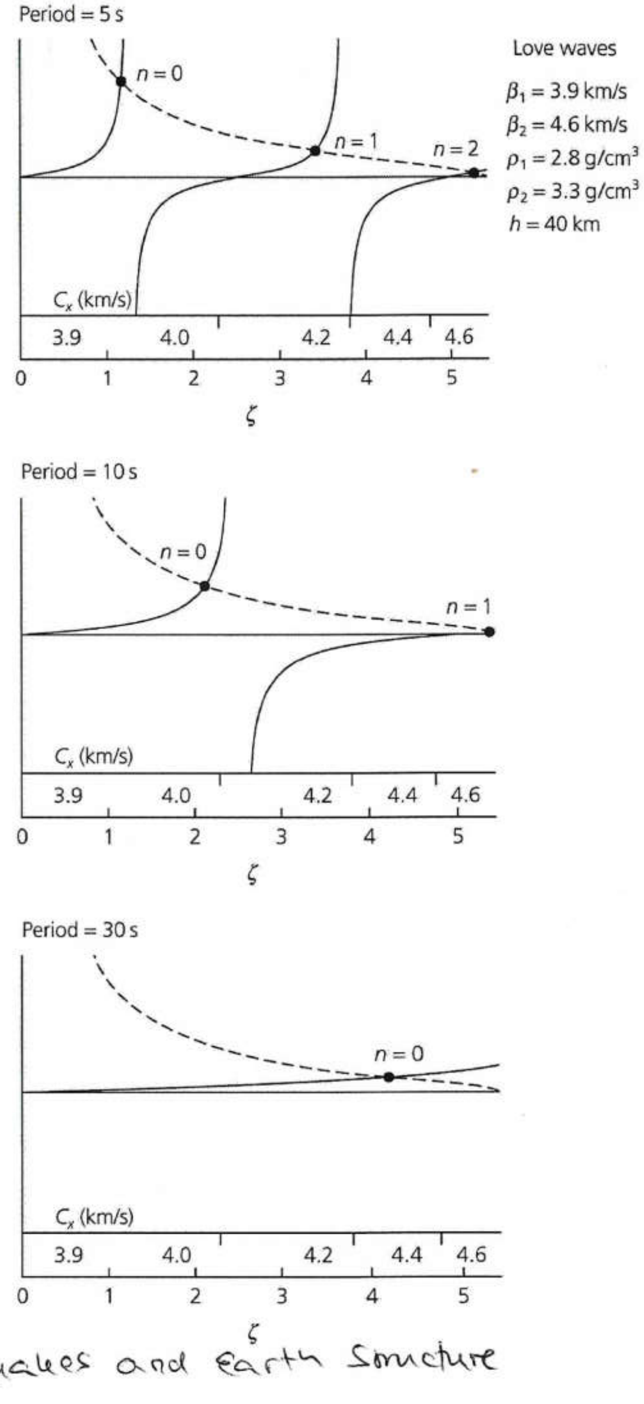
Define $\quad \zeta = (h/c_x)(c_x^2/\beta_1^2 - 1)^{1/2}$ to get:

$$\tan(\omega\zeta) = \left(\frac{\mu_2(1 - c_x^2/\beta_2^2)^{1/2}}{\mu_1}\right)\left(\frac{h}{c_x\zeta}\right)$$

Fundamental modes: $n = 0$

Overtone modes: $n > 0$

igure 2.7-8: Solution of the dispersion relation for Love waves i layer over a halfspace.



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30) Za disperzivne valove izvedite sljedeće relacije:

a) $u = c - \lambda \frac{dc}{d\lambda}$

b) $u = c^2 \frac{dT}{d\lambda}$

c) $u = -\lambda^2 \frac{df}{d\lambda}$

$T = \frac{\lambda}{c}$

$k = \frac{2\pi}{\lambda}$

$c = \frac{\omega}{k}$

$f = \frac{\omega}{2\pi}$

a) $u = \frac{d\omega}{dk} \quad / \cdot \frac{dc}{d\omega}\frac{dk}{d\lambda}$

$$\frac{dc}{d\omega}\frac{dk}{d\lambda} u = \frac{d\omega}{dk}\frac{dc}{d\omega}\frac{dk}{d\lambda}$$

$c = \frac{\omega}{k}$

$u = \frac{d\omega}{dk}$

trebaju nam ove jednadžbe:

$$\frac{dk}{d\lambda} = \frac{d}{d\lambda}\left(\frac{2\pi}{\lambda}\right) = -\frac{2\pi}{\lambda^2} = -\frac{k}{\lambda}$$

$$\frac{dc}{d\omega} = \frac{d}{d\omega}\left(\frac{\omega}{k}\right) = \frac{1}{k} - \frac{\omega}{k^2}\frac{dk}{d\omega} = \frac{1}{k} - \frac{c}{ku}$$

$$\left(\frac{1}{k} - \frac{c}{ku}\right)\left(-\frac{k}{\lambda}\right) u = \frac{dc}{d\lambda}$$

$$u = c - \lambda \frac{dc}{d\lambda}$$

b) $u = \frac{d\omega}{dk} \quad / \cdot \frac{dk}{d\lambda}\frac{dT}{d\omega}$

$$\frac{dT}{d\omega} = \frac{d}{d\omega}\left(\frac{2\pi}{\omega}\right) = -\frac{2\pi}{\omega^2} = -\frac{T}{\omega}$$

$$\frac{df}{d\omega} = \frac{d}{d\omega}\left(\frac{\omega}{2\pi}\right) = \frac{1}{2\pi}$$

$$\frac{dk}{d\lambda}\frac{dT}{d\omega} u = \frac{d\omega}{dk}\frac{dk}{d\lambda}\frac{dT}{d\omega}$$

$$\left(-\frac{k}{\lambda}\right)\left(-\frac{T}{\omega}\right) u = \frac{dT}{d\lambda}$$

$$u = \frac{\omega}{k}\frac{\lambda}{T}\frac{dT}{d\lambda} \Rightarrow u = c^2 \frac{dT}{d\lambda}$$

c) $u = \frac{d\omega}{dk} \quad / \cdot \frac{dk}{d\lambda}\cdot\frac{df}{d\omega}$

$$\frac{dk}{d\lambda}\frac{df}{d\omega} u = \frac{d\omega}{dk}\frac{dk}{d\lambda}\frac{df}{d\omega} \Rightarrow \left(-\frac{k}{\lambda}\right)\left(\frac{1}{2\pi}\right) u = \frac{df}{d\lambda}$$

$$u = \lambda\left(-\frac{2\pi}{k}\right)\frac{df}{d\lambda} \Rightarrow u = -\lambda^2 \frac{df}{d\lambda}$$